Early-Stage Entrepreneurial Financing: A Signaling Perspective

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Abstract

We analyze an entrepreneur's choice between angel and venture-capital (VC) financing given staged financing in a competitive investment market. The key to our analysis is the idea that a negative signal is inferred by the market if a previous investor chooses not to follow on a subsequent investment. We show that in a benchmark situation where ventures are ex-ante identical, entrepreneurs retain higher ownership shares by financing with angel investors who can commit to a single round. When entrepreneurs are ex-ante heterogeneous, there is a separating equilibrium where entrepreneurs with higher (lower) likelihoods of success finance with VC firms (angel investors).

Keywords: Entrepreneurial financing; insider signaling; venture capital; angel investing

JEL Classifications: G14, G24, D82

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1 Introduction

The market for early-stage investments has grown dramatically in the past decade. Angel investing, which used to be a boutique practice partaken by some successful entrepreneurs, has become a mass market with hundreds of angel syndicates and platforms dedicated to make small investments in early-stage startups.[1] In contrast, traditional VCs are said to have moved to larger and later stages of financing and tend not to invest in deals that seek less than three or four million dollars (OECD, 2011; Sohl, 2011). Moreover, it is commonly understood that angel financing is thus chosen by entrepreneurs because VC financing is simply unavailable in the early stages of the firm (e.g., Hellmann and Thiele, 2014).

This explanation, however, is incomplete, because VC firms have put in place dedicated funds to make small, seed investments, viewing them as sources for potential follow-on larger investments. In fact, there are now hundreds of VC funds (such as those run by Andreessen Horowitz) that behave like angels in the early stages of investment. Hence, if one seeks to understand the choice between angel and VC financing in the early stages of a venture, it is crucial to gain insight into the different dynamics that play out in a model of staged financing. In particular, our focus in this paper is on the idea that a first-stage financier who decides not to re-invest in a venture at a later stage conveys a negative signal to outsiders.

We take explicit account of an entrepreneur's reluctance to cede additional ownership shares to financiers in exchange for financing. That is, our model incorporates entrepreneurs' tradeoffs in financing on favorable terms and sharing the ownership of a business.[2] This is consistent with the pecking-order theory (e.g., Myers and Majluf, 1984), which says that entrepreneurs seek financing in an order that minimizes ownership dilution. For instance, entrepreneurs would prefer to borrow from banks rather than to sell equity stakes, everything else being equal. As is commonly the case, however, for a penniless entrepreneur or an early-

[1] A taxonomy for these early-stage investors includes 'super angels' (e.g., Ron Conway, Peter Thiel, etc.), 'micro-VCs' (e.g., First Round Capital, True Ventures, SoftTech VC, etc.), and 'startup accelerators' (Y Combinator, TechStars, Seedcamp, etc.).

[2] Anecdotes abound in the popular press about the importance of share dilution in a founder's decision to raise capital. There is also growing evidence that there are indeed substantial non-pecuniary benefits for founder-CEOs (see, e.g., Hamilton, 2000; Moskowitz and Vissing-Jørgensen, 2002).

stage venture, debt financing is often unavailable.

We thus focus on how an entrepreneurs' ownership incentives influence his choice of a financing source in the early financing stage of his venture, assuming that both angels and VCs provide competitive equity financing. Angel investors, who are characterized by a liquidity constraint (due to, e.g., lack of larger follow-on funds), cannot participate in the subsequent round of investment, whereas VCs are characterized by having the option to decide whether or not to re-invest. Importantly, a VC firm's second-round participation decision is observed by outside investors, which enables the market to partially update its posterior beliefs about a venture's likelihood of success.

More specifically, if a seed investor decides to re-invest, then a positive signal is sent to the market, which ramps up outside investors' valuation of the venture; if the seed investor decides to stay out of a subsequent round, then a negative signal is sent and the venture's valuation decreases. Because the re-investment decision leads to more competitive outside offers, a decision to re-invest ratchets up the expected value of the venture. This endogenously creates an 'up' round with a higher valuation (or a 'down' round with a lower valuation) than what would have been observed without such strategic incentives, and gives rise to a threshold above which the seed investor will follow through.

We first show that given a competitive capital market, when ventures are ex-ante identical, entrepreneurs can retain higher ownership stakes by financing the early rounds with angel investors. Intuitively, the signaling problem that follows VC financing in first-round investments creates uncertainty associated with the inside VC's participation decision in the second round. This uncertainty leads to an overall smaller expected ownership share for the entrepreneur compared to angel financing. This benchmark result (i.e., ownership advantage with angel financing) holds regardless of whether outside investors are willing to invest or the venture is simply liquidated in a 'down' round (where inside VCs do not follow through).

When entrepreneurs are heterogeneous with respect to (i) ex-ante private information about their ventures' viability, and (ii) the cost of accessing investors, a separating equilibrium exists, where entrepreneurs with high likelihoods of success (high types) tend to choose VC financing in the first stage, while low types choose angel financing. That is, our model can explain the co-existence of angels and VCs in the early stage and gives rise to the prediction that those who finance with angel investors are of lower average quality.

The remainder of the paper is organized as follows. Section 2 discusses the relevant literature. Section 3 presents the model, and Section 4 analyzes VC financing. Section 5 compares an entrepreneur's expected ownership share under VC and angel financing. Section 6 considers the case of heterogeneous entrepreneurs. Section 7 examines robustness to interim liquidation. Section 8 concludes. All formal proofs are in the Appendix.

2 Related Literature

There is a growing literature that examines angel financing. Kerr et al. (2014) show that angels engage in efficient selection and screening processes just as traditional VCs do. However, angels do not manage large pools of capital, and thus entrepreneurs often have to secure follow-on investments from VC firms. In fact, angels typically do not follow on when a subsequent financing round involves VC participation (Wong et al., 2009).[3] Practitioners' writings suggest that VCs who are dedicated to early-stages basically think of their portfolios as priority access to potential investment leads or optionalities. Hence, our main focus is on the signaling hypothesis rather than different characteristics of angels and VCs per se.

We are not the first to consider the lack of re-investment capacity of angel investors. As previously mentioned, Hellmann and Thiele (2014) analyze the complementary role that angels and VC firms play in the process of financing new ventures in a search and matching model. A main difference is that in our model VCs can also participate in the early stage of financing, so that entrepreneurs have a choice between angels and VCs. Chemmanur and Chen (2014) compare these two methods of financing based on their value-adding capacity, whereby VC firms invest in industries where their potential for adding value is large and

[3]Additionally, in a 2010 survey of angel investors in Europe, the number of deals made by respondents was 331, 222, and 127 in France, Italy, and the UK, respectively, of which the number of follow-on rounds was only 2, 7, and 4 (EBAN, 2010).

angels fund firms in industries where value-adding potential is more limited. While we think this is a complementary view, our focus here is on the insider-signaling problem, where share dilution, rather than moral hazard, is the primary issue.

There is, in fact, a large literature on competition between inside and outside financiers (particularly, banks) under asymmetric information. However, existing work mostly assumes that the outside banks learn nothing about the firm before bidding against the inside bank. An exception is Rajan (1992), where a public signal that is correlated with the inside bank's private information is realized, so that the outside banks update their beliefs before bidding on re-investment terms. The subtle difference is that in our model, there does not exist an exogenous public signaling device as in Rajan (1992); rather, the private information is signaled endogenously via the inside VC's follow-on decision itself. That is, outside investors update their beliefs because they can observe the inside VC's action (through, e.g., interviews or investment networks), rather than through an exogenous revelation of a public signal.

As a broader class, signaling models are ubiquotous in the literature. For instance, an entrepreneur with a promising venture can signal the value of his venture through his willingness to retain equity (e.g., Leland and Pyle, 1977; Gale and Stiglitz, 1989; Courteau, 1995). The difference is that in our model, the signal is generated by financiers at the interim stage — as a result of their re-investment decisions, whereas in the aforementioned signaling models the signal is generated by entrepreneurs who possess ex-ante private information. We connect to this literature by considering both types of signaling in our model, and show that there are separating equilibria in which entrepreneurs trade off the advantages from both ex-ante and interim signaling.

Finally, our paper is related to the VC-financing literature (e.g., Admati and Pfleiderer, 1994; Gompers, 1995; Neher, 1999; Bergemann and Hege, 2005), where the focus is mostly on agency problems in the relationship between entrepreneurs and financiers. To the extent that VCs spend more time than angels on guiding entrepreneurs in the early stages (which may or may not be the case), entrepreneurs could prefer VC financing. However, the literature has largely neglected the important consideration of ownership tradeoffs across financing sources.

Our model adds that entrepreneurs can retain larger ownership shares by choosing angels, and also larger ownership stakes can help mitigate moral-hazard problems in early-stage ventures.

3 The Model

Penniless entrepreneurs seek to finance their ventures. Entrepreneurs are assumed to be risk neutral and ventures are ex-ante identical (we later relax this assumption). The representative entrepreneur has preferences defined over his ownership share of the venture and the value of his equity. Specifically, his utility increases in both expected ownership share and value of retained equity. This means that the entrepreneur is willing to trade off a (small) decrease in equity value for an increase in his ownership share of the venture. We do not need to assume anything specific about the functional form of the utility function or the degree of substitutability between its two components. All of our results hold as long as there is some (even arbitrarily small but positive) substitutability between ownership share and equity value.[4]

At the beginning of the game, nature draws a venture idea for each entrepreneur, which is characterized by a probability of success p. We assume that p is uniformly distributed between 0 and 1 in order to derive closed-form solutions. Venture development consists of two stages. In the first stage, an entrepreneur raises capital K to turn his idea into a prototype. No revenue is generated at this stage. Hence, if the venture is liquidated at the end of the first round, then the firm's equity is worthless. In the second stage, the entrepreneur raises growth capital F. At the end of the second stage, the venture either succeeds with probability p generating a revenue R, or fails with probability $1 - p$ yielding zero revenue. We assume that ventures have ex-ante positive net values, that is, $R/2 \geq (K + F)$.

To raise capital in each stage, entrepreneurs approach risk-neutral investors. The invest-

[4] As previously mentioned, the main advantage of angel financing in our model is that entrepreneurs retain larger ownership shares. If entrepreneurs only cared about the value of their equity, then it can be shown that VC financing is as good as or preferred to angel financing.

ment market is perfectly competitive at both stages of financing, so that (new) investors can only expect to earn a zero rate of return at the time of investment. This assumption reflects the recent evidence that most VC funds did not outperform public equities in the 2000s (e.g., Harris et al., 2014). Direct evidence on the performance of angel investments is rare, but given the estimates that the angel market and venture-capital market are roughly equal in size (e.g., Ibrahim, 2008), the perfect competition assumption can serve as a useful benchmark and allows us to present our results in a clear manner.

There are ex ante two types of investors, to which we refer as 'VCs' and 'angels.' The difference between the two is that angel investors are those who commit not to re-invest in follow-on rounds for reasons that are exogenous to the model (e.g., lack of funds or portfolio policies) while VCs have the option of deciding whether or not to re-invest. This distinction largely matches how the inside signaling problem with VCs is perceived in practice. The investor type can be endogenized by assuming an ex-ante probability of a liquidity constraint, but this does not change any of the results in our model, although it can make closed-form analysis intractable.[5]

A venture's success likelihood, p, is initially unknown to both the entrepreneur and investors. If an investor invests in the first round, the seed investor as well as the entrepreneur learn the venture's success likelihood, p. Information asymmetry arises because p is not observed by outside investors. However, before second-round bidding begins, outside investors can observe whether the seed investor decides to make a follow-on investment, and investors update their beliefs before finalizing offers. We later allow the entrepreneur to have private information before the first round begins, but the information propagation in terms of the interim signal observed by outside investors in the second stage remains throughout.

Both angel investors and VCs offer equity financing.[6] Specifically, in each stage, investors offer a contract to the entrepreneur, (γ_X, X), which gives the investor a share $\gamma_X \in (0, 1)$

[5]See Supplementary Material at the end of this manuscript for review.

[6]Given the lack of an agency problem, we abstract from convertible securities because the cash flows are equivalent to those of equity financing in our model. Specifically, the zero-profit condition ensures that any discount or conversion ratio applied to convertibles yields a zero rate of return.

of the venture's equity in return for investing X, where $X \in \{K, F\}$, and the entrepreneur accepts the best financing offer. As a tie-breaking rule, we assume that the entrepreneur stays with the seed investor when indifferent. The tie-breaking rule does not affect our analysis, but it captures the fact that the seed-round contract may specify a *right of first refusal*. That is, seed investors have the right to make follow-on investments as long as they can match the financing terms offered by the market.

Finally, we assume that the seed investor's follow-on decision is irreversible. That is, seed investors cannot initially decline to participate in the follow-on round in an attempt to later beat the terms of (undervalued) outside offers. Anecdotes suggest that such maneuvers can lead to reputational damages for VC firms in the investor community, and the literature often makes this assumption (e.g., Admati and Pfleiderer, 1994). The assumption is also consistent with the evidence that VCs do not create down rounds in order to dilute the entrepreneur's ownership shares (e.g., Broughman and Fried, 2012). The timing of the game is summarized as follows:

1. Entrepreneur raises a first financing round of size K.
2. Seed investor learns venture's success probability, p.
3. Seed investor announces its second-round participation decision.
4. Outside investors update their valuation and make financing offers.
5. Entrepreneur raises a second financing round of size F, and profits are realized.

4 VC Financing

The solution concept we employ is pure-strategy Perfect-Bayesian Equilibria. As is standard in the signaling literature, given that the venture quality distribution is atomless, we focus attention on equilibria characterized by binary threshold rules.[7] Specifically, we consider

[7] As is often the case in the context of threshold equilibria, restricting attention to pure strategies is without loss of generality (see, e.g., Milgrom and Weber, 1985).

the following strategy for inside VCs in the second round: make the follow-on investments if and only if the realization of the venture's success likelihood is above a certain threshold p^*. Because the VC's profit increases continuously in the probability p, by construction this strategy yields a unique equilibrium.

Given such a strategy, outside investors update their beliefs about the venture's success likelihood based on the inside VC's follow-on decision. Conditional on the VC following through, the venture's expected probability of success is given by $\frac{1+p^*}{2}$, while it is $\frac{p^*}{2}$ when the inside VC decides not to participate in the second round. Hence, for $p^* < 1$, we have $\frac{p^*}{2} < \frac{1}{2} < \frac{1+p^*}{2}$, whereby the venture's second-round valuation either increases (an 'up' round), if the VC follows through, or decreases (a 'down' round) relative to its first-round valuation, if the VC decides not to re-invest.

Thus, there are two types of second-round subgames depending on the model's parameterization. In the second stage, a venture has a non-negative expected value if and only if $pR \geq F$. If the realization of success probabilities were perfectly observed, then no venture with success probability below $\frac{F}{R}$ would be financed in the second round. Under asymmetric learning, however, the expected probability of success conditional on a down round is $\frac{p^*}{2}$. If $\frac{p^*}{2}$ is greater than $\frac{F}{R}$, then outside investors would find it worthwhile to invest in a down round even if the inside VC decided not to re-invest.

The inside VC may not find it worthwhile to re-invest in ventures that turn out to be marginal because its participation sends a positive signal to the market, which leads to more competitive outside offers. Thus, if the venture turns out to be only moderately promising, the inside VC firm lets outside investors finance the venture's follow-on round. Hence, the expected value of a venture in a down round remains positive and the venture may still attract additional funding F. To be precise, the following condition ensures that ventures in a down round are financed by outside investors:

Assumption 1. $4KR \leq (R - 2F)^2$.

Because the left-hand side is a linear function of R and the right-hand side is a quadratic function of R, holding the capital requirements (K and F) constant, Assumption 1 is trivially satisfied for a large payoff, R. Assumption 1 also guarantees that the equilibrium threshold value p^* is at the interior $(0,1)$, so that both up and down rounds can be observed. We assume Assumption 1 in the proceeding analysis, but we also demonstrate in Section 7 that our results continue to hold when ventures are instead liquidated in a down round.

Working backwards, the financing terms in an up round are characterized by the zero-profit condition, $\frac{1+p^*}{2}\gamma_F R - F = 0$, whereby the required share will be bid down to $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, and the entrepreneur remains with the inside VC. Similarly, the financing terms in a down round will be bid down to $\gamma_F^- = \frac{2F}{p^* R}$. The inside VC's first-round share is diluted in both cases. Specifically, if a VC owns a share γ_K of equity following the first round, then this share will become $\gamma_K(1-\gamma_F)$ after percentage dilution following the second round. Hence, the inside VC will finance the second round if and only if p satisfies

$$p(\gamma_K(1-\gamma_F^+) + \gamma_F^+)R - F \geq p(\gamma_K(1-\gamma_F^-))R. \tag{1}$$

The left-hand side of (1) is the expected profit from making the follow-on investment. The inside VC's share is the sum of the first-stage share γ_K diluted by a factor of $(1-\gamma_F^+)$, and the newly acquired share γ_F^+. The right-hand side is the VC's expected profit if it decides not to follow through and let outside investors finance the second round, where the outsiders' equity share is specified by γ_F^-.

Proposition 1 *There is a threshold value p^*, above which the inside VC re-invests in the second round in exchange for an equity share of $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, and below which an outside investor finances the venture in exchange for an equity share of $\gamma_F^- = \frac{2F}{p^* R}$.*

The equilibrium displays properties that are consistent with the so-called 'signaling hypothesis' of VC seed investments. That is, if the early-stage VCs do (not) follow through, then it sends a positive (negative) signal to the market. In particular, the second-round financing terms only depend on the seed investor's participation, despite the probability of success p being known to the inside VC and to the entrepreneur.

We can now derive the first-stage share γ_K^* of the seed investor. The seed investor would expect to participate in the second round with probability $1 - p^*$ and stay out of it with probability p^*. Notice that the fallback position (a down round) yields a positive expected profit under Assumption 1, as long as its initial share γ_K is positive. Given the competitive investment market, the financing terms (γ_K^*, K) for the first-round are characterized by the usual zero expected-profit condition (we note here, however, that the seed investor's ex-post realized returns can be positive if the venture succeeds):

$$-K + (1-p^*)\left(\frac{1+p^*}{2}(\gamma_K(1-\gamma_F^+) + \gamma_F^+)R - F\right) + p^*\left(\frac{p^*}{2}(\gamma_K(1-\gamma_F^-))R\right) = 0. \quad (2)$$

Substituting in for γ_F^+, γ_F^-, and p^* and simplifying give our next result.

Proposition 2 *The equilibrium equity share of the VC firm investing in the first round is* $\gamma_K^* = \frac{2K}{R-2F}$. *The equilibrium threshold likelihood of success,* p^*, *above which the VC firm will follow through, is given by* $p^* = 1 - \frac{4K}{R-2F}$.

5 Angel Financing

We now consider a class of investors, to which we refer as angels, that specializes in early stage investments — whether due to lacking sufficient funds to follow through or because of a stated policy of exclusively investing in early-stage ventures. As a consequence of financing a seed stage with an angel, the entrepreneur must raise capital in the second-round from outside investors. Suppose that, in the first stage, an entrepreneur receives offers from angels as well as from VCs. There are two possible outcomes. First, the entrepreneur may accept an offer from a VC firm, in which case the subsequent equilibrium is described by Propositions

1 and 2 above. Second, the entrepreneur may accept a financing offer from an angel in the first round.

In the latter case, consider the second-stage subgame. Since angels do not re-invest, outside investors do not glean any new information from angels' non-participation. Hence, the same ex-ante information environment as in the first round applies to outside investors, who would then bid down the equity share until the expected profit from the investment is driven down to zero, that is, $\frac{1}{2}\gamma_F R - F = 0$. Thus, an outside investor's second-round equity share is specified by $\gamma_F^0 = \frac{2F}{R}$. Similar to the preceding section, the angel investor's initial share, $\hat{\gamma}_K$, is determined by a zero expected-profit condition which takes into account the subsequent percentage dilution of $(1 - \gamma_F^0)$.

Let $(\hat{\gamma}_K, K)$ denote the first-round terms offered by angels and let (γ_K^*, K) denote the first-round terms offered by VCs. If an entrepreneur chooses angel financing in the first round, then the second-round terms are given by (γ_F^0, F); if he instead chooses VC financing, then the second-round terms are given by (γ_F^-, F) with probability p^* and (γ_F^+, F) with probability $1 - p^*$. Thus, the entrepreneur's ownership share is $1 - [\hat{\gamma}_K(1 - \gamma_F^0) + \gamma_F^0]$ under angel financing, compared to an expected ownership share of $1 - [\gamma_K^*(1 - E\gamma_F) + E\gamma_F]$, where $E\gamma_F = p^*\gamma_F^- + (1 - p^*)\gamma_F^+$, under VC financing. The following proposition establishes the benchmark result.

Proposition 3 *An entrepreneur's expected ownership share is larger with angels than with VC financing, and entrepreneurs prefer to finance with angels in the first stage.*

The intuition for this result is as follows. The ex-ante surplus created by the venture is unaffected by the information revelation at the interim stage because the venture is always funded in the second round, but the equity required by financiers are inversely related (hence convex) to the venture valuation. The share of the equity given up is larger under VC financing (with two possible valuations) than under angel financing (with a single valuation) because of Jensen's Inequality ($E\gamma_F > \gamma_F^0$). Although the initial equity shares exchanged in the first round are the same under VC and angel financing ($\hat{\gamma}_K = \gamma_K^*$), the expected second-round equity payment is thus larger under VC financing.

On the other hand, it is straightforward to see that the entrepreneur's expected equity *value* is the same regardless of his first-round financing choice because (i) the expected venture value is fixed ex ante, and (ii) given a perfectly competitive investment market, the ex-ante expected cost of financing ends up the same for angels and VCs. For instance, the value of equity given up in the second round following angel financing is $E[\gamma_F pR] = F$, and the value of equity given up following inside VC financing is $E[\gamma_F pR] = p^*\gamma_F^- E[p|p < p^*]R + (1-p^*)\gamma_F^+ E[p|p > p^*]R = 2F\frac{p^*}{2} + (1-p^*)F = F$. It can also be shown that the first-stage equity cost analogously equals the investment K.

The observation that the entrepreneur's expected equity value is the same across angels and VCs suggests that the inside VC's signaling possibility is not necessarily a concern *ex ante* for entrepreneurs. However, as Proposition 3 indicates, the entrepreneur's financing decision may still be affected by insider signaling, since his expected ownership share is lower under VC financing. Recalling that entrepreneurs' utility encompasses both share value and share size, with a tradeoff between equity value and ownership share, Proposition 3 then implies that entrepreneurs in this benchmark case would in fact strictly prefer to finance the first round with angels — because, in expectation, they retain a larger equity stake while equity value is equal.

Contingent Contracts:

One may wonder whether the above result is due to the fact that the VC firm's first-round terms, (γ_K^*, K), did not depend on second-round contingencies (i.e., up or down rounds). In the following, we show that the above result is robust to the introduction of contingent contracts. First, notice that restricting attention to equity financing in the second round is without loss of generality, because various forms of claims are indistinguishable given a cash flow realization, R. Second, in the case of angel financing, financing terms are not in fact contingent on any event since angels do not create up or down rounds in the second stage.

A VC firm's first-round financing terms can be made contingent on the creation of an up or down round in the second stage. That is, the first-round financing terms can be $(\gamma_K^+, \gamma_K^-, K)$, where γ_K^+ and γ_K^- are the VC's shares contingent on an up or down round, respectively. In this case, the subgame as well as overall equilibria are still characterized by equations (1) and (2) above. The only change is that now there is a range of financing contracts consistent with these two conditions (rather than the unique solution we characterized above). Formally, the VC firm follows through when the success likelihood exceeds a threshold,

$$p^*(\gamma_K^+(1-\gamma_F^+) + \gamma_F^+)R - F \geq p^*(\gamma_K^-(1-\gamma_F^-))R, \tag{3}$$

and the profits from the first-round participation must be zero,

$$-K + (1-p^*)\left(\frac{1+p^*}{2}(\gamma_K^+(1-\gamma_F^+) + \gamma_F^+)R - F\right) + p^*\left(\frac{p^*}{2}(\gamma_K^-(1-\gamma_F^-))R\right) = 0. \tag{4}$$

Given the financing terms for the second-round investment (γ_F^+ and γ_F^-), equations (3) and (4) define three equilibrium quantities $(\gamma_K^+, \gamma_K^-, p^*)$, which leaves one degree of freedom. VC firms are, however, indifferent among the set of contingent-claims contracts that are consistent with (3) and (4) due to the zero expected profit condition. The remaining question is then whether there are any contingent-claims contracts that VCs can offer in the first round, which leave entrepreneurs with larger expected shares. The following proposition shows that this is not the case.

Proposition 4 *Suppose VC firms can offer contingent-claims contracts* $(\gamma_K^+, \gamma_K^-, K)$ *in the first stage. Proposition 3 continues to hold.*

The intuitive reason is that by using contingent contracts VCs cannot eliminate the signaling problem. Hence, our results establish the ownership advantage of angel financing in the early stages of a venture. We note here that if VCs were to include anti-dilution protection in a down round, then it would only make VC contracts less desirable; hence, entrepreneurs would not accept VC contracts with anti-dilution protection in our model given competing offers. This is also consistent with empirical evidence that full ratchet anti-dilution is only found in less than 10% of VC contracts (Bengtsson and Sensoy, 2011).

However, we acknowledge that the perfect competition assumption is restrictive. For instance, entrepreneurs may prefer value added by VC firms *even in the early stages*. While we view value added and inside signaling as relatively orthogonal issues to analyze, a real-world choice may well depend on these broader tradeoffs. For instance, in order to finance with a reputable VC firm, entrepreneurs may sign a VC contract that includes protective clauses at worse terms, which shield the VC from outside competition in the follow-on rounds. Hence, we caution that our benchmark results require careful interpretation.

6 Ex-Ante Heterogeneity and Private Information

Thus far, we have assumed that ventures are ex-ante identical; that is, a venture's probability of success is initially unknown by all parties. Signaling occurs at the second (interim) stage, after the entrepreneur and the inside financier learn about this probability. Moreover, we assumed that both VCs and angels are equally accessible by entrepreneurs. In this section, we relax these assumptions by (i) considering the case where entrepreneurs possess (ex-ante) private information, and (ii) considering an asymmetric access cost where entrepreneurs incur a higher cost of approaching VCs. We then demonstrate the existence of a separating equilibrium where angel and VC financing co-exist in the first stage.

Suppose that at the beginning of the game, an entrepreneur privately learns his venture's likelihood of success p with probability $\theta \in (0, 1)$, and he learns nothing with probability $1 - \theta$. (Alternatively, a fraction θ of entrepreneurs are *informed* and a fraction $1 - \theta$ are *uninformed*.) It is useful to discuss another benchmark where all entrepreneurs are privately informed about their venture's likelihood of success (i.e., $\theta = 1$). Notice that this setup entails its own signaling problem for entrepreneurs *in the first stage*. That is, entrepreneurs can try to signal their ventures' type with their choice of financiers.

Let p^* denote the marginal type above which VC firms would follow through, and consider first the case where all entrepreneur types pool on VC financing. It is well known that there are multiple equilibria in such signaling setups due to the fact that equilibrium consistency imposes no restrictions on off-equilibrium beliefs, so an equilibrium refinement is required to eliminate less desirable equilibria. We refine our set of equilibria by imposing the Intuitive Criterion (Cho and Kreps, 1987) for those subgames that occur with probability zero in equilibrium.[8]

To be precise, off-equilibrium beliefs are specified by a type distribution on $[0, p^*)$ for an entrepreneur who deviates to angel financing. The refinement entails that a deviation comes from the type that would benefit the most from doing so. If an entrepreneur deviates from VC pooling to angel financing, those entrepreneur types who stand most to benefit are those seeking to avoid the VC's down round, hence $p < p^*$. Without this refinement, for instance, an angel pooling equilibrium can be supported by less plausible off-equilibrium beliefs (e.g., entrepreneurs who deviate to VC financing have $p = 0$). The following proposition describes the unique equilibrium that survives this refinement criterion.

Proposition 5 *Suppose all entrepreneurs privately learn their ventures' types with probability $\theta = 1$. Then all entrepreneurs choose VCs financing in the first stage.*

Proposition 5 demonstrates that angel financing unravels in equilibrium when entrepreneurs are ex-ante perfectly informed about their ventures' types. To see this, consider a threshold equilibrium, with a cutoff value $\tilde{p} \in (0, 1)$, such that entrepreneurs with $p < \tilde{p}$ finance the first-round with angels, and those with $p \geq \tilde{p}$ finance with VCs (the opposite can also be considered, but trivially cannot be sustained because of the signaling nature of VC financing). Then the former type can strictly gain by deviating from angel financing to VC financing, even if they know that they will face a down round, because the VC terms are based on more favorable beliefs, that is, $\tilde{p} \leq p < p^*$. Consequently, when $\theta = 1$, financiers glean no new

[8] Technically, the refinement requires the set of entrepreneurs' types to be finite. We can take the set of entrepreneur types to be an arbitrarily large set of finite numbers in the interval $[0, 1]$.

information about the ventures' success probabilities by observing the entrepreneur's choice of first-stage financiers.

Intuitively, the difference from the previous section is the presence of adverse selection, whereby there is no mechanisms for the high types to sustain separation from the low types in the first stage. To gain some element of separation, high types would flock to VC financing, where they benefit from separation due to insider signaling in the second stage. However, since the equilibrium involves complete unraveling, the outcome has the flavor of a prisoner's dilemma situation. In particular, from Proposition 3, we know that entrepreneurs are better off (they retain larger equity shares) by financing with angels. Thus, if commitment and coordination were possible, then all entrepreneurs would be better off by committing entirely to angel financing prior to learning about their ventures' types.

Separating Equilibrium:

Consider now the extended model, where $\theta \in (0, 1)$. Notice that the equilibrium characterized in Proposition 5 continues to hold, given the aforementioned off-equilibrium beliefs. The only additional requirement to establish the VC pooling equilibrium is to verify that the *uninformed* entrepreneurs (with mass $1 - \theta$) would not benefit by deviating to angel financing. This is trivially satisfied because even informed types who know that they will face a down round with VCs have no incentive to switch to angels. Since uninformed types are not guaranteed to have a down round, it is straightforward to check that they too possess no incentives to deviate from VC pooling.

However, the VC pooling equilibrium characterized above is, of course, inconsistent with real-world observations, where angel investments have grown substantially and represent a considerable segment of the market for early-stage financing. Here, we show that there are in fact separating equilibria where, among the informed group of entrepreneurs, relatively low-type ventures finance with angels and relatively high-type ventures finance with VCs. To show this, we consider a plausible scenario where entrepreneurs are also differentiated by their ability to approach investors; that is, some entrepreneurs may incur higher costs of doing so than others due to locational advantages or preexisting networks.

Let us consider a continuous distribution over the possible costs for entrepreneurs of accessing VCs (denoted by c), with a cumulative distribution function $G(c)$ with support on $[0, \bar{c}]$. We assume that the upper bound $\bar{c}$ is sufficiently large so that some uninformed types choose to finance with angels in equilibrium. If $\bar{c}$ is restricted to a small number, then no uninformed type may choose angels, but we still obtain a separating equilibrium among the informed. Below, we illustrate the former, more interesting case. We normalize the cost of accessing angels to 0, although that need not be the case for our qualitative results to hold.[9]

To gain some intuition for why a separating equilibrium can be sustained, suppose that all *informed* entrepreneurs (mass θ) finance with VCs, and all *uninformed* entrepreneurs (mass $1 - \theta$) finance with angels. For any given θ, the first-stage beliefs are the same as the uniform prior (i.e., there is no belief updating) for both groups of entrepreneurs. Given that VCs and angels finance the same distribution of ventures, an informed entrepreneur with a type p arbitrarily close to zero would strictly benefit by switching to angel financing — because by so doing, investors' beliefs are minimally affected (i.e., financing terms are arbitrarily close to being unchanged), yet the entrepreneur avoids a down round under VC financing.

Building on the above, a deviation by a mass of low-type ventures to angel financing would "weigh down" the first-stage beliefs of angel investors, but improve the first-stage beliefs of VC investors. That is, as more informed entrepreneurs with low p types switch to angels, the first-stage belief of VC investors is increasingly more favorable than that of angel investors. There then exists a threshold informed $\tilde{p}$ type, apart from the VC access costs, who is indifferent between angel and VC financing; that is, for the threshold type, the benefit from avoiding a down round from VC financing in the second stage exactly offsets the loss due to facing less favorable first-stage beliefs.

Then, with a positive cost for accessing VCs, informed types above $\tilde{p}$ would also pursue

[9]We note that the costs for accessing VCs are distributed independently from the distribution on venture types. Assuming negative correlation, i.e., that informed entrepreneurs who are more likely to succeed also incur lower access costs only strengthens our results.

angel financing if their VC-access costs are high. We show in the proof that the aforementioned threshold $\tilde{p}$ type satisfies $\tilde{p} < p^*$; that is, the threshold type will face a down round in the second stage under VC financing. This observation has two direct implication. First, *uninformed* entrepreneurs have a chance at an up round given VC financing; moreover, all ventures face equal terms from angels. Thus, the uninformed types strictly prefer VC financing if their access costs are low. Second, informed types with $p \geq p^*$ are guaranteed an up round under VC financing and are discontinuously made less sensitive to VC-access costs.[10] The above observations are summarized in Figure 1.

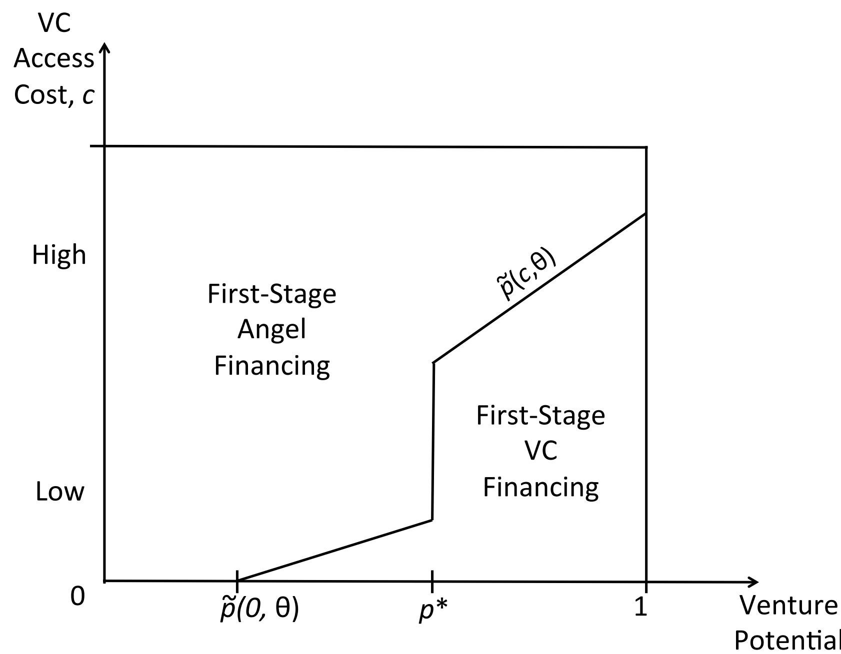

Figure 1: First-round investor choice by informed types for a given θ.

Proposition 6 *For any $\theta \in (0,1)$ and realizations of $c \in [0, \bar{c}]$, there exists a threshold value $\tilde{p}(c, \theta)$ such that informed entrepreneurs for whom $p < \tilde{p}(c, \theta)$ finance with angels and those with $p \geq \tilde{p}(c, \theta)$ finance with VCs. Uninformed entrepreneurs with low (high) access costs finance with VCs (angels).*

[10] More specifically, the access cost threshold above which the informed p^* type would switch to angel financing exhibits a "jump" relative to informed types slightly below p^*.

Proposition 6 establishes that angels and VCs draw different sets of entrepreneurs in the early stage. The exact equilibrium characterization of course varies with the model's parameter specification, but the qualitative features of these separating equilibria are essentially the same. First, in the early rounds, the distribution of ventures that receive VC financing first-order stochastically dominates the distribution of ventures that finance with angels. Second, given a non-negligible cost of accessing VCs, angels tend to finance a larger portion of early-stage rounds. Third, it may be in the interest of VCs to maintain high access costs but to a degree, because doing so helps attract an increasingly more favorable, but also smaller, segment of early-stage ventures.

7 Interim Liquidation

Below we discuss robustness of our results to Assumption 1. If Assumption 1 is not satisfied, then ventures will be liquidated in a down round, generating no revenues. Notice that at the interim stage (the second round), outside investors would be only willing to participate if they can at least break even. This is the case if and only if a venture's expected value is non-negative, i.e., $E[pR - F|h] \geq 0$, where h denotes a history of the game observed by outside investors. Thus, the conditions under which outside investors are willing to participate in the second round, following VC financing in the first round, is as follows.

- **Up round:** $E[p|p \geq p^*] = \frac{1+p^*}{2} \geq \frac{F}{R}$. Simplifying yields

$$RF - F^2 + 2KR \leq (R - F)^2. \tag{5}$$

- **Down round:** $E[p|p < p^*] = \frac{p^*}{2} \geq \frac{F}{R}$. Simplifying yields

$$4KR \leq (R - 2F)^2. \tag{6}$$

Notice that (6) is Assumption 1 in the previous analysis. This condition can be re-written as $2RF + 3F^2 + 4KR \leq (R - F)^2$; hence, Assumption 1 implies that (5) is satisfied. This is intuitive because the posterior beliefs for ventures in a down round are worse than for those in an up round. Notice that somewhere in between these two conditions lies the condition for angel investors to participate in the first round, $R \geq 2F + 2K$. Previously, Assumption 1 guranteed $R \geq 2F + 2K$ trivially; however, if we only assume here that inequality (5) is satisfied, then $R \geq 2F + 2K$ becomes a binding assumption.

In the following, we characterize the equilibrium when inequality (5) is satisfied but (6) is not. That is, we assume $RF - F^2 + 2KR \leq (R - F)^2 < 2RF + 3F^2 + 4KR$, to which we refer as the Limited Viability (LV) case, and show that our main results continue to hold with a slightly more binding assumption (i.e., $R > 3F + 2K$) as a *sufficient* condition. This is indeed stronger than the basic assumption, $R \geq 2F + 2K$. However, we think that most ventures would have a sufficiently large expected revenue $R/2$ relative to the required capital $(F + K)$, so that this sufficient condition is not so restrictive.

Similarly to the previous analysis, let p^*_{LV} denote the inside VC's threshold value, whereby the VC will follow through if and only if the realized probabailitiy of success is greater than p^*_{LV}; otherwise, the venture is liquidated in the second stage. Outside investors' updated beliefs are that the success likelihood of a venture conditional on the inside VC following through is uniform on $[p^*_{LV}, 1]$. Thus, the financing terms in an up round are given by $\gamma_F^+ = \frac{2F}{(1+p^*_{LV})R}$. Since ventures in a down round are worth nothing, the inside VC's outside option value is now zero. An inside VC would re-invest if and only if the following condition is satisfied:

$$p(\gamma_K(1 - \gamma_F^+) + \gamma_F^+)R - F \geq 0.$$

Then, for a given γ_K, the optimal threshold value p^*_{LV} is determined by

$$p^*_{LV}(\gamma_K + (1 - \gamma_K)\frac{2F}{(1 + p^*_{LV})R})R - F = 0. \tag{7}$$

Ex ante, the second-round terms are $\gamma_F^+ = \frac{2F}{(1+p_{LV}^*)R}$ with probability $1 - p_{LV}^*$ and non-investment with probability p_{LV}^*. Thus, the expected profit of a first-round investor can be written as

$$-K + (1 - p_{LV}^*)\left(\frac{1+p_{LV}^*}{2}(\gamma_K + (1-\gamma_K)\frac{2F}{(1+p_{LV}^*)R})R - F\right) \geq 0.$$

Setting this expected profit equal to zero, substituting in for (7), and simplifying yield

$$F(1 - p_{LV}^*)^2 = 2Kp_{LV}^*.$$

Solving for p_{LV}^*, the only root satisfying $p_{LV}^* \leq 1$ is given by

$$p_{LV}^* = \frac{F + K - \sqrt{2FK + K^2}}{F}.$$

One can then substitute p_{LV}^* into (7) to solve for the equilibrium first-round terms (γ_K^*, K).

Proposition 7 *With limited-viability ventures, entrepreneurs' expected ownership shares are larger with angels than with VC financing if* $R > 2K + 3F$.

Proposition 7 means that when the entrepreneurs are ex-ante identical (as in Section 3's model), the ownership advantage associated with angel financing continues to hold even if ventures are liquidated in a down round. Consequently, entrepreneurs prefer financing with angels in the first stage.

It can be further shown that the equilibrium characterization in Propositions 5 and 6 (where entrepreneurs have private information with probability θ and incur VC-access costs) extends to this section under more limited circumstances. Specifically, while the VC pooling equilibrium always exists (when there are no VC-access costs), the separating equilibrium may fail to exist if the mass of uninformed entrepreneurs who pursues angel financing is too small (i.e., if θ is sufficiently high or if VC-access costs are relatively low).

To gain some intuition, let us first consider whether the VC-pooling equilibrium continues to exist when there are no VC-access costs. The analogous off-equilibrium beliefs (i.e., for angel financing) under VC pooling in the LV case are distributed on $[0, p^*_{LV})$. Notice that these beliefs entail liquidation (i.e., no financing) for those entrepreneurs who seek angel investment. Hence, there is no incentive to deviate from VC financing, and the VC-pooling equilibrium continues to exist.[11]

Second, consider the separating equilibrium from the previous section when VC-access costs are positive. For any set of parameter values, we observe that $\tilde{p} \geq p^*_{LV}$ must be satisfied, where $\tilde{p}$ is the threshold type below which informed entrepreneurs with no VC-access costs choose angel financing (for a given set of parameter values). To see this, suppose, by way of contradiction, that the candidate separating equilibrium satisfies $\tilde{p} < p^*_{LV}$. Notice then that informed entrepreneurs in $[\tilde{p}, p^*_{LV})$ would strictly benefit by switching from VC to angel financing — because otherwise their ventures will be liquidated in the second stage — a contradiction.

Consider then a candidate separating equilibrium where $\tilde{p} \geq p^*_{LV}$, which otherwise satisfies the same condition as the separating equilibrium delineated in Section 6. Then those informed entrepreneurs with types $p < p^*_{LV}$ are already choosing angels, and no further deviation need take place.

Finally, notice that for a candidate equilibrium with $\tilde{p} \geq p^*_{LV}$ to exist, the mass of the uninformed group who chooses angels must be sufficiently large. To see this, consider the limiting case where the mass of the uninformed group is arbitrarily small. Then the threshold value $\tilde{p}$ is arbitrarily close to zero, violating the condition $\tilde{p} \geq p^*_{LV}$. Therefore, given any set of parameter values, a separating equilibrium can exist in the LV case, with both types of investors offering financing, provided that the mass of the uninformed group who seeks angel financing is sufficiently large (i.e., θ is sufficiently small and/or VC-access costs are sufficiently high).

[11] Technically, informed entrepreneurs with $p < p^*_{LV}$ are indifferent about participating, but their participation in VC financing is necessary to sustain equilibrium.

8 Conclusion

This paper examined informational aspects of the choice between angel and VC financing in the early stages of a venture. When outside investors evaluate a venture, they are typically at an informational disadvantage relative to inside investors; however, they may still be able to update their beliefs by observing an inside investor's re-investment decision. Anticipating greater share dilution if the inside VC does not follow through, an entrepreneur's expected ownership share is lower with VC financing than with angel financing even in a competitive investment market. This acts in favor of angels if entrepreneurs care about ownership as well as equity.

When entrepreneurs have access to private information, another layer of signaling is introduced, pertaining to entrepreneurs' choice of first-round financiers. While a VC-pooling equilibrium always exists, there is a separating equilibrium that can help explain the co-existence of angels and VCs in the early financing stages. Specifically, the separating equilibrium gives rise to the prediction that angels finance entrepreneurs with no private information and those having relatively low probabilities of success; and VCs tend to finance ventures with relatively high probabilities of success. These findings may shed some light on the dynamics behind early-stage entrepreneurial financing.

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9 Appendix

Proposition 1

Proof. Let the outside VC firm's strategy be to offer to finance the second round in return for acquiring a $\gamma_F^+ = \frac{2F}{(1+p^*)R}$ share in an up round and a $\gamma_F^- = \frac{2F}{p^*R}$ in a down round, respectively. We can easily see that this strategy profile satisfies the zero-profit condition. First, in an up round, the expected probability of success is $\frac{1+p^*}{2}$, so that an outside firm's expected profit is $\frac{1+p^*}{2}\gamma_F^+ R - F = 0$. Since the offer terms are the same, the entrepreneur stays with the inside investor by the tie-breaking assumption. Second, in a down round, the expected probability of success is $\frac{p^*}{2}$, so that an outside investor's expected profit is $\frac{p^*}{2}\gamma_F^- R - F = 0$. Since only outside investors offer to finance, the entrepreneur takes any one of such offers.

We now verify that the inside VC firm's cut-off strategy is optimal given outside firms' strategy. When the inside VC decides to follow through, outside VCs believe that the success probability is greater than p^*; and, when it does not, the belief is $p < p^*$. It is never optimal for the inside VC to offer better terms than the market; and offering worse terms does no better because outside VCs will offer better terms, which the inside VC has to match. Given this, it is optimal for the inside VC firm to finance an entrepreneur in the second round if and only if the investor's expected profit conditional on $p > p^*$ is greater than its expected profit conditional on $p \leq p^*$, that is, if and only if the observed success probability p satisfies the following inequality:

$$p(\gamma_K + (1-\gamma_K)\frac{2F}{(1+p^*)R})R - F \geq p(\gamma_K(1 - \frac{2F}{p^*R}))R.$$

Setting both sides equal and simplifying yields the threshold value, $p^* = 1 - 2\gamma_K$. ■

Proposition 3

Proof. Notice that if an angel invests in the first round, then there is no signal sent to the market at the beginning of the second round. Given our Assumption 1, outside VC firms will invest in the second round following angel investment (with no signal from an inside investor) because the expected probability of success is higher than in a down round. The second-round investor's share, $\gamma_F^0 = \frac{2F}{R}$, is pinned down by the zero-profit condition. Given this, an angel investor's expected profit from the first-round investment is $-K + \int_0^1 p(\gamma_K(1-\frac{2F}{R})R)dp$, where the second term is the expected share value. The angel investor's share is derived by use of a zero-profit constraint, $-K + \frac{1}{2}\gamma_K(1-\frac{2F}{R})R = 0$, which gives $\hat{\gamma}_K = \frac{2K}{R-2F}$.

The entrepreneur's share, if he finances with an angel in the first round, is $1-[\hat{\gamma}_K(1-\gamma_F^0)+\gamma_F^0]$. On the other hand, if the entrepreneur finances with a VC firm that can re-invest in the second round, his expected share is $1-[\gamma_K^*(1-E\gamma_F)+E\gamma_F]$, where $E\gamma_F = p^*\gamma_F^- + (1-p^*)\gamma_F^+$, from the previous section. The entrepreneur's ownership share is larger with an angel than with a VC firm if and only if $\hat{\gamma}_K(1-\gamma_F^0) + \gamma_F^0 < \gamma_K^*(1-E\gamma_F) + E\gamma_F$. By substitution, $\hat{\gamma}_K(1-\gamma_F^0)+\gamma_F^0 = \frac{2K+2F}{R}$. Since $E\gamma_F = p^*\gamma_F^- + (1-p^*)\gamma_F^+ = p^*\frac{2F}{p^*R} + (1-p^*)\frac{2F}{(1+p^*)R} = \frac{2F}{(1-\gamma_K^*)R}$, it follows that $\gamma_K^*(1-E\gamma_F) + E\gamma_F = \gamma_K^* + \frac{2F}{R}$. Thus, the inequality holds if and only if $\frac{2K+2F}{R} < \frac{2K}{R-2F} + \frac{2F}{R}$, or $\frac{2K}{R} < \frac{2K}{R-2F}$, which is indeed the case. ■

Proposition 4

Proof. Setting $\gamma_F^+ = \frac{2F}{(1+p^*)R}$ and $\gamma_F^- = \frac{2F}{p^*R}$ and solving equations (3) and (4) with equality, we obtain

$$\gamma_K^+ = \frac{F(1-p^*)p^* + 2K(1+p^*)}{(1+p^*)R - 2F}$$

and

$$\gamma_K^- = \frac{2Kp^* - F(1-p^*)^2}{p^*R - 2F}. \tag{8}$$

Proposition 3 then holds if and only if

$$\hat{\gamma}_K(1-\gamma_F^0) + \gamma_F^0 < p^*(\gamma_K^-(1-\gamma_F^-) + \gamma_F^-) + (1-p^*)(\gamma_K^+(1-\gamma_F^+) + \gamma_F^+).$$

Using (3) and substituting $\hat{\gamma}_K(1-\gamma_F^0)+\gamma_F^0=\frac{2K+2F}{R}$, we obtain

$$\frac{2K+2F}{R} \le p^*(\gamma_K^+(1-\gamma_F^+)+\gamma_F^+ +\gamma_F^-)-\frac{F}{R}+(1-p^*)(\gamma_K^+(1-\gamma_F^+)+\gamma_F^+).$$

Rearranging yields

$$\frac{2K+3F}{R} \le \gamma_K^+(1-\gamma_F^+)+\gamma_F^+ + p^*\gamma_F^-.$$

Substituting for γ_K^+, γ_F^-, γ_F^+, and simplifying, Proposition 3 continues to hold when

$$\frac{2K+3F}{R} \le \frac{2K+4F-Fp}{R},$$

which is true since $p^* \le 1$. ■

Proposition 5

Proof. Suppose there exists a threshold $\tilde{p} \in (0,1)$ such that entrepreneurs with ideas $p \ge \tilde{p}$ finance the first round with a VC firm and entrepreneurs with ideas $p < \tilde{p}$ finance with an angel. Let p^*, $p^* \ge \tilde{p}$, denote the threshold level above which the VC firm follows through. Then the second-stage zero profit condition, $E[p|p>p^*]\gamma_F^+ R - F = 0$, determines $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, and similarly $E[p|\tilde{p} < p \le p^*]\gamma_F^- R - F = 0$ determines $\gamma_F^- = \frac{2F}{(\tilde{p}+p^*)R}$. In the following, let γ_K^+ denote the VC firm's share from the first round of financing given the threshold strategies. Then γ_K^+ is obtained from

$$-K + \frac{1-p^*}{1-\tilde{p}}\left(\frac{1+p^*}{2}(\gamma_K^+ + (1-\gamma_K^+)\frac{2F}{(1+p^*)R})R - F\right)$$

$$+\frac{p^*-\tilde{p}}{1-\tilde{p}}\left(\frac{\tilde{p}+p^*}{2}\gamma_K^+(1-\frac{2F}{(\tilde{p}+p^*)R})R\right) = 0.$$

Rearranging and simplifying yields

$$\gamma_K^+ = \frac{2K}{(1+\tilde{p})R - 2F}.$$

Given the VC firm's strategy, an entrepreneur with $p \leq p^*$ who finances with a VC will have an ownership share of

$$(1 - \gamma_K^+)(1 - \gamma_F^-), \tag{9}$$

because the entrepreneur knows that he will face a down round in the second stage. Similarly, an entrepreneur with $p > p^*$ who finances with a VC knows that he will face an up round and thus has an ownership share of $(1 - \gamma_K^+)(1 - \gamma_F^+)$, which is larger than (9) because $\gamma_F^+ < \gamma_F^-$.

On the other hand, entrepreneurs with $p < \tilde{p}$ finance with angels. In the second round, γ_F^0 is bid down such that $\frac{\tilde{p}}{2}\gamma_F^0 R - F = 0$, giving $\gamma_F^0 = \frac{2F}{\tilde{p}R}$. Let γ_K^0 denote here the angel's first-round equity share in exchange for K. Then γ_K^0 is obtained from the zero-profit condition, $\frac{\tilde{p}}{2}\gamma_K^0(1 - \frac{2F}{\tilde{p}R})R - K = 0$, giving $\gamma_K^0 = \frac{2K}{\tilde{p}R - 2F}$.

If $\tilde{p}$ is below a certain value, then $\gamma_K^0 > 1$, which means that the angel investor cannot break even by investing in the venture given that only entrepreneurs with $p < \tilde{p}$ apply to angels. Thus, given a sufficiently low threshold value for $\tilde{p}$, all entrepreneurs have to finance with VCs, resulting in the VC pooling equilibrium.

If $\tilde{p}$ is above a certain value, then it becomes possible to have $\gamma_K^0 \leq 1$, so that angel investors can break even. In this case, an entrepreneur with $p < \tilde{p}$ who finances with an angel will have an ownership share of

$$(1 - \gamma_K^0)(1 - \gamma_F^0). \tag{10}$$

First, suppose an entrepreneur with $p < \tilde{p}$ deviates to VC financing. Comparing (9) and (10), the entrepreneur would gain from such a deviation because $\gamma_K^+ < \gamma_K^0$ and $\gamma_F^- < \gamma_F^0$. That is, the entrepreneur's share is higher under VC financing. Notice that the entrepreneur knows his success likelihood p, so his equity value is also higher given that his equity share increases. Therefore, the threshold $\tilde{p}$ cannot be positive.

Second, suppose all entrepreneurs finance with VCs, that is, $\tilde{p} = 0$. From Propositions 1 and 2, the VC firm's shares in such a case are $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, $\gamma_F^- = \frac{2F}{p^*R}$, and $\gamma_K^* = \frac{2K}{R-2F}$. A deviation to an angel investor (a probability zero event) would lead to $\gamma_F^0 = \frac{2F}{p^*R}$ and $\gamma_K^0 =$

$\frac{2K}{p^*R-2F}$, given that the off-equilibrium beliefs is $p \in [0, p^*)$. The deviating entrepreneur's equity share as well as value are lower because $\gamma_F^0 = \gamma_F^-$ and $\gamma_K^0 > \gamma_K^*$. Thus, no entrepreneur has an incentive to deviate.

Third, consider the case in which entrepreneurs with $p \geq \tilde{p}$ finance with angels and those with $p < \tilde{p}$ finance with VCs. Then the VC's shares ars $\gamma_F^+ = \frac{2F}{(\tilde{p}+p^*)R}$ and $\gamma_F^- = \frac{2F}{p^*R}$. Using a zero-profit condition, the VC's first-round share is $\gamma_K^+ = \frac{2K}{\tilde{p}R-2F}$. On the other hand, an angel investor's shares are $\gamma_K^0 = \frac{2K}{(1+\tilde{p})R-2F}$ and $\gamma_F^0 = \frac{2F}{(1+\tilde{p})R}$. Following the same logic as above, an entrepreneur with $p \leq \tilde{p}$ possesses a profitable deviation by instead financing with an angel because $\gamma_K^+ > \gamma_K^0$ and $\gamma_F^- > \gamma_F^+ > \gamma_F^0$.

Fourth, suppose all entrepreneurs finance with angels. Then the angel investor's shares are $\gamma_K^0 = \frac{2K}{R-2F}$ and $\gamma_F^0 = \frac{2F}{R}$. A deviation to VC financing leads to $\gamma_K^+ = \frac{2K}{(1+\tilde{p})R-2F}$ and $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, given the off-equilibrium beliefs $p \in [p^*, 1]$. An entrepreneur with $p > p^*$ knows that he will face an up round in the second stage with a VC. Such an entrepreneur is indeed better off by deviating to VC financing because $\gamma_F^0 > \gamma_F^+$ and $\gamma_K^0 > \gamma_K^+$, i.e., because his equity share as well as share value increase. ■

Proposition 6

Proof. Suppose for a moment that there are no VC-access costs and that all uninformed entrepreneurs finance with angels. For a given $\theta \in (0, 1)$, suppose there exists a threshold $\tilde{p}(\theta)$ (where we use $\tilde{p}(\theta)$ to refer to $\tilde{p}(c = 0, \theta)$) such that informed entrepreneurs with $p < \tilde{p}(\theta)$ finance with angels and those with $p \geq \tilde{p}(\theta)$ finance with VCs. Then the expected type of an entrepreneur who finances with an angel is given by

$$p_0(\theta) = \frac{\theta\tilde{p}(\theta)}{1-\theta+\theta\tilde{p}(\theta)}\frac{\tilde{p}(\theta)}{2} + \frac{1-\theta}{1-\theta+\theta\tilde{p}(\theta)}\frac{1}{2} = \frac{1-\theta+\theta\tilde{p}(\theta)^2}{2(1-\theta+\theta\tilde{p}(\theta))}.$$

With a slight abuse of notation, it follows that the angel financing terms are $\gamma_F^0 = \frac{F}{p_0(\theta)R}$ and $\gamma_K^0 = \frac{K}{p_0(\theta)R-F}$ as previously shown. Thus, an entrepreneur's equity share under angel financing at the end of the second round is given by $(1-\gamma_K^0)(1-\gamma_F^0)$.

Similarly, a VC's financing terms are $\gamma_F^+ = \frac{2F}{(1+p^*)R}$, $\gamma_F^- = \frac{2F}{(\tilde{p}(\theta)+p^*)R}$, $\gamma_K^+ = \frac{2K}{(1+\tilde{p}(\theta))R-2F}$, where $p^*(\tilde{p})$ is defined by the VC's indifference condition for re-investment in the second round.

For a given θ, there is a mass θ of informed entrepreneurs and a mass $1-\theta$ of uninformed entrepreneurs, both uniformly distributed on $[0,1]$. First, notice that there exists a threshold type $\tilde{p}(\theta)$ such that informed entrepreneurs with $p < \tilde{p}(\theta)$ finance with angels and those with $p \geq \tilde{p}(\theta)$ finance with VCs.

To see this, suppose by way of contradiction that $\tilde{p}(\theta) = 0$, whereby all informed entrepreneurs finance with VCs and all uninformed entrepreneurs finance with angels. Consider informed entrepreneurs whose types fall below the threshold for an up round under VC financing. It follows from Proposition 3 that $(1-\gamma_K^0)(1-\gamma_F^0) > (1-\gamma_K^+)(1-\gamma_F^-)$, i.e., such informed entrepreneurs would prefer to pursue angel financing.

Suppose alternatively that $\tilde{p}(\theta) = 1 - \frac{1-\sqrt{1-\theta}}{\theta}$. Then it can be shown that $p_0(\theta) = \tilde{p}(\theta)$ as well as $\frac{\partial \tilde{p}(\theta)}{\partial \theta} < 0$. Consider the informed entrepreneur with $p < 1 - \frac{1-\sqrt{1-\theta}}{\theta}$. His share when financing with an angel is $(1-\gamma_K^0)(1-\gamma_F^0)$, where $\gamma_F^0 = \frac{F}{\tilde{p}(\theta)R}$ and $\gamma_K^0 = \frac{K}{\tilde{p}(\theta)R-F}$. If he finances with a VC, his share is $(1-\gamma_K^+)(1-\gamma_F^-)$, where $\gamma_F^- = \frac{2F}{(\tilde{p}(\theta)+p^*)R}$ and $\gamma_K^+ = \frac{2K}{(1+\tilde{p}(\theta))R-2F}$. Because $\gamma_F^0 > \gamma_F^-$ and $\gamma_K^0 > \gamma_K^+$, the entrepreneur will deviate to VC financing. Similarly, by considering a $\tilde{p}(\theta)$ that is arbitrarily close to p^*, it is straightforward to see that the informed $\tilde{p}(\theta)$ type would prefer to switch to VC financing, a contradiction.

Notice that $p_0(\theta)$ is a continuous function of $\tilde{p}(\theta)$. It follows from the Intermediate Value Theorem (utility function given angel and VC financing have to cross at least once) that there exists a value $\tilde{p}(\theta) \in (0, 1 - \frac{1-\sqrt{1-\theta}}{\theta})$ such that informed entrepreneurs in $p \in [0, \tilde{p}(\theta))$ finance with angels and those in $p \in [\tilde{p}(\theta), 1]$ finance with VCs, and no informed entrepreneurs have an incentive to deviate.

An analysis that is analogous to the above applies given any mass of uniformly distributed entrepreneurs on $[0,1]$ that finances with angels, and under any cost c of accessing VC financing. Notice that such a mass exists because the upper bound of the support of the distribution on VC-access costs is assumed to be sufficiently high, and the density function is positive on the support. In particular, uninformed entrepreneurs whose VC-access cost realizations are high are uniformly distributed and will pursue angel financing. All remaining arguments are in the text. ■

Proposition 7

Proof. Substituting p^*_{LV} in for (7) and rearranging yield

$$\gamma_K = \frac{F^2\left(\sqrt{K(2F+K)}-K\right)}{\left(\sqrt{K(2F+K)}-F-K\right)\left(2F^2+R\sqrt{K(2F+K)}-2FR-KR\right)}.$$

Since ventures are liquidated in a down round with probability p^*_{LV}, the entrepreneur's expected equity share is given by

$$(1-p^*_{LV})(1-\gamma^+_F)(1-\gamma_K).$$

Substituting in for $\gamma^+_F = \frac{2F}{(1+p^*_{LV})R}$, we have

$$(1-p^*_{LV})(1-\frac{2F}{(1+p^*_{LV})R})(1-\gamma_K).$$

From Proposition 3, the entrepreneur's expected share under angel investing is given by $1-\frac{2K+2F}{R}=\frac{R-2(K+F)}{R}$. Thus, the entrepreneur's shares are larger with angels if

$$\frac{R-2(K+F)}{R} > (1-p^*_{LV})(1-\frac{2F}{(1+p^*_{LV})R})(1-\gamma_K).$$

Substituting in for p^*_{LV} and simplifying, this inequality can be rewritten as

$$\frac{R-2(K+F)}{R} > \frac{\sqrt{2FK+K^2}-K}{F}(1-\frac{2F^2}{(2F+K-\sqrt{2FK+K^2})R})(1-\gamma_K).$$

Substituting in for γ_K and simplifying, this further reduces to

$$F^2 + (R - F)(R - 2K - 3F) > 0.$$

A sufficient condition is thus given by $R > 2K + 3F$. $\blacksquare$

10 Supplementary Material (For Review)

As mentioned in footnote 5, the ex-ante distinction between VCs and angels is without loss of generality. This is because an entrepreneur's expected ownership share is monotone increasing in the likelihood of the inside VC's liquidity constraint in the second stage. That is, in Section 3's model, even if investors have a continuum of probabilities for future liquidity constraints, the investor who will be liquidity-constrained with certainty leaves the entrepreneur with the largest expected ownership share. Said another way, there is no intermediate investor type (i.e., with a probability in $(0, 1)$ of a liquidity constraint in the second stage) that gives a higher (lower) expected ownership share than the investors having the extreme 0 (1) likelihood of a liquidity constraint.

Specifically, rather than assuming the two types of investors (VCs and angels) ex ante, suppose now that first-round investors face an idiosyncratic liquidity shock prior to the second round. Let $\lambda_i \in [0, 1]$ denote the probability that the capital requirement F does not violate an inside investor i's future liquidity. With probability $1 - \lambda_i$, the investor does not have enough liquidity and hence is unable to participate. We assume, more realistically, that the investor's ex-ante type, λ_i, is public information, but the realization of the random variable (i.e., whether or not the investor ends up facing a liquidity constraint in the second stage) is the investor's private knowledge.

To characterize the equilibrium of this model variant, we proceed in steps similar to those taken in our earlier analysis. If the first-round investor re-invests, then the financing terms will be $\gamma^+_{F,i} = \frac{2F}{(1+p^*_i)R}$ (in an up round), where p^*_i denotes the threshold above which investor i follows through. If the insider does not re-invest, then unlike in the previous analysis there are now two possibilities. One is that the inside investor has the liquidity but chose not to follow on, and the other is that the first-round investor does not in fact have the necessary liquidity due to an idiosyncratic shock.

Notice that in the latter event, the inside investor's decision not to participate, had it been distinguishable, would not convey a negative signal. However, the realization of the investor's liquidity shock is private information. Thus, outside investors' updated beliefs are given by $\lambda_i \frac{p_i^*}{2} + (1-\lambda_i)\frac{1}{2}$, and the equity share $\gamma_{F,i}^-$ for an outside investor (in a down round) following a first-round investor of type type λ_i is $\gamma_{F,i}^- = \frac{2F}{(1-\lambda_i(1-p_i^*))R}$. Notice that if $\lambda_i = 1$, then $\gamma_{F,i}^- = \frac{2F}{p^*R}$ as in the base model's case of 'VC' financing; if $\lambda_i = 0$, then $\gamma_{F,i}^- = \frac{2F}{R}$, as in the case of 'angel' financing.

Intuitively, when the first-round investor's probability of possessing the requisite liquidity is small, the signal transmitted in a down round does not reflect as poorly on the venture. Accordingly, both the entrepreneur and the first-round investor sustain less dilution in a down round compared to the case where the investor has the necessary liquidity. This creates a complex, nonlinear equilibrium relationship between λ_i and the inside investor's equity share $\gamma_{K,i}$ in the first round. Hence, a part of the proof of the following proposition is based on numerical simulations.

Proposition 8 *The share of the first-round equity that is given up has an inverse U-shaped relation to the liquidity shock λ_i. The entrepreneur's expected ownership share is largest when the investor has $\lambda_i = 0$.*

Proof. The threshold venture type p_i^* above which a first-round investor of type λ_i follows through, if it has the requisite liquidity in the second stage, is determined by the following indifference condition:

$$p_i^*(\gamma_{K,i} + (1-\gamma_{K,i})\frac{2F}{(1+p_i^*)R})R - F = p_i^*\gamma_{K,i}(1 - \frac{2F}{(1-\lambda_i(1-p_i^*))R})R,$$

from which we obtain

$$p_i^* = \frac{2\lambda_i(1-\gamma_{K,i}) - 1 + \sqrt{1 + 4\gamma_{K,i}(2 - \lambda_i(3 - \lambda_i\gamma_{K,i}))}}{2(2\gamma_{K,i}(1-\lambda_i) + \lambda_i)}. \tag{11}$$

Investor i's first-round equity share, $\gamma_{K,i}$, obtained in exchange for K, is defined by the following zero expected profit condition:

$$-K + \lambda_i\big[(1-p_i^*)(\frac{1+p_i^*}{2}(\gamma_{K,i} + (1-\gamma_{K,i})\gamma_{F,i}^+)R - F) + p_i^*\frac{p_i^*}{2}\gamma_{K,i}(1-\gamma_{F,i}^-)R\big] \tag{12}$$

$$+(1-\lambda_i)\big[\frac{1}{2}\gamma_{K,i}(1-\gamma_{F,i}^-)R\big] = 0$$

where $\gamma_{F,i}^+ = \frac{2F}{(1+p_i^*)R}$, $\gamma_{F,i}^- = \frac{2F}{(1-\lambda_i(1-p_i^*))R}$, and p_i^* is specified by (11). Notice that it follows from (12) that when $\lambda_i = 0$ or 1, $\gamma_{K,i} = \frac{2K}{R-2F}$ consistent with the base model.

From the standpoint of an entrepreneur, his expected equity share is given by

$$(1-\gamma_{K,i})\big[\lambda_i(1-p_i^*\gamma_{F,i}^- - (1-p_i^*)\gamma_{F,i}^+) + (1-\lambda_i)(1-\gamma_{F,i}^-)\big].$$

The rest of the statements are based on numerical simulations. ■

The following figures plot a typical simulation run, where the parameter values are $R = 100$ and $K = F = 10$.

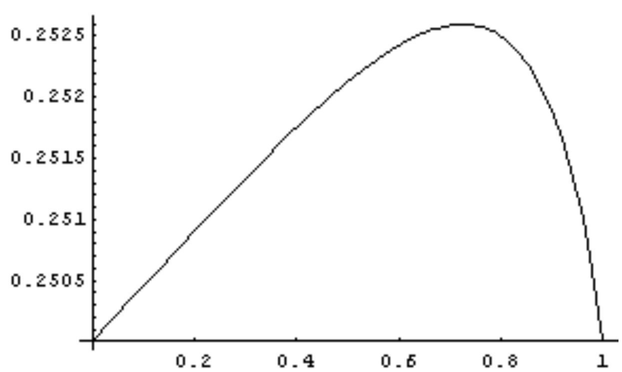

Figure 2: First-round equity share $\gamma_{K,i}$ as a function of λ_i.

As Figure 2 shows, the first-round equity share $\gamma_{K,i}$ is non-monotonic in λ_i, reaching a peak at an intermediate level of λ_i. The intuitive reason is as follows. The liquidity shock facilitates uncertainty regarding whether the first-round investor can re-invest or not. Similarly to the base model, where uncertainty regarding the first-round investor's follow-on decision facilitated expected share dilution for entrepreneurs, uncertainty regarding the

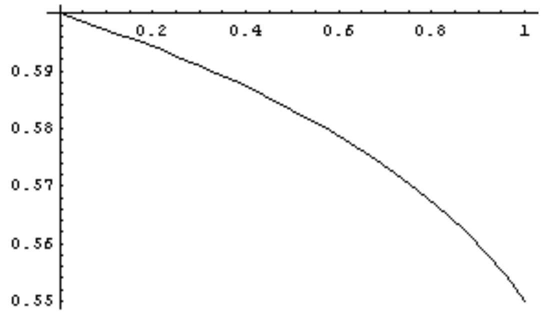

Figure 3: Entrepreneur's expected share as a function of λ_i.

insider's liquidity increases the initial investor's expected share dilution. This, in turn, leads the first-round investor to require a greater equity share in the first round, which becomes particularly noticeable as λ_i moves away from the extremes.

Figure 3 plots an entrepreneur's expected equity share as a function of λ_i for the same set of parameter values as above. The entrepreneur's expected share is monotone decreasing in λ_i, so that his share is largest when the first-round investor is guaranteed to be liquidity-constrained ($\lambda_i = 0$). We ran simulation experiments over a wide range of parameter values for R, K, and F, and found the same pattern. Thus, the ownership advantage associated with angel financing (when all entrepreneurs are ex-ante identical) appears robust to the introduction of liquidity shocks. This shows that our main results do not rely on the extreme categorization of angels and VC investors.