Strategic Obfuscation of Production Capacities

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Abstract

Recent supply-chain models that study competition among capacity-constrained producers omit the possibility of producers strategically setting wholesale prices to create uncertainty with regards to (that is, to obfuscate) their production capacities. To shed some light on this possibility, we study strategic obfuscation in a supply-chain model comprised of two competing producers and a retailer, where one of the producers faces a privately-known capacity constraint. We show that capacity obfuscation can strictly increase the obfuscating producer's profit, therefore presenting a clear incentive for such practices. Moreover, we identify conditions under which both producers' profits increase. In effect, obfuscation enables producers to tacitly collude and charge higher wholesale prices by moderating competition between producers. The retailer, in contrast, suffers a loss in profit, raises retail prices, while overall channel profits decrease. We show that the extent of capacity obfuscation is limited by its cost and by a strategic retailer's incentive to facilitate a deterrence.

Keywords: Obfuscation, Capacity Constraints, Strategic Manipulation, Supply Chain, Pricing.

1 Introduction

The Sherman Act, enacted in 1890, condemns any explicit agreements among competitors to fix prices, rig bids, allocate customers, or engage in anticompetitive behavior as unlawful. Law scholars interpret such agreements as "*a negotiation that concludes when the firms convey mutual assurances that the understanding they reached will be carried out*" [3]. In practice, tacit collusive behavior, which skirts legal restrictions, can arise in many forms — from price leadership, where a dominant firm sets general industry prices with other firms following suit, to firms engaging in unilateral communications to the market about future conduct (e.g., via press releases or other media forms), enabling other market actors to align their strategies.

Consider California and Washington states, for example, where oil refineries sell gasoline to gas-station operators who subsequently sell it to end consumers. Watchdog organizations and lawmakers have recently accused oil refineries of employing anti-competitive practices by artificially raising wholesale prices and creating a perception of supply shortages, when in fact refineries possessed additional production capacities. Providing validation to such claims are studies on refinery emissions data that show that the price spikes observed in May and October 2012, for instance, happened when gasoline inventories actually increased [23]. The report concludes that it is very possible — if not probable — that the sudden and jarring jump in the price of gasoline in California was the result of collusive behavior by the refineries.

In this paper, we seek to understand the impact that strategic shortages (or the appearance of shortages) may have on competitors' actions in a supply chain. We propose a model where a producer may strategically choose to price less aggressively in order to alter perceptions about his privately known capacity; that is, take actions to obfuscate his true capacity. The question we ask is the following: What are the net effects of capacity obfuscation on the producer's profits, competitors' profits, channel profits, and consumer surplus? In our model, a retailer sells two competing, though possibly differentiated product brands, brand i and brand j. Through the choice of a wholesale price, the producer of brand j may affect his competitor's (producer i's) beliefs about his capacity, and by doing so, producer j may induce his competitor to pursue a less aggressive pricing strategy.

The incentive to obfuscate capacity emerges in many settings. In cargo shipping, freight forwarding companies (retailers), which rent containers to end consumers, contract with shipping companies (producers) to transport containers to their destinations. While the maximum capacity of each shipping fleet is known, actual operating capacity is private information. Shipping companies are able to obfuscate their capacities by strategically slowing ship speeds, and this allows them to quote higher prices. Collectively, this practice allows shipping companies to negotiate better contracting terms with forwarding companies.[1] Strategic capacity withholding (leading to collusive pricing by firms) has also been documented in the electricity market literature [19, 28].[2] In the electricity market, energy generators (representing producers) engage in auctions operated by independent system operators (retailers), who then sell power to local utility companies (representing end consumers). Borenstein et al. [5] find significant departures from competitive pricing in California using data from June 1998 to October 2000, and identify that 20% of markups were due to producers capturing some competitive rents. Wolak and Patrick [35] describe similar practices in the U.K. electricity market, where higher-cost combined-cycle gas turbine generators owned by new entrants provided base-load power that could have been supplied more cheaply by coal-fired plants that withheld output capacity (and were owned by two of the largest producers). In broader terms, Tellidou and Bakirtzis [33] use agent-based simulations to show the likely emergence of tacit collusion in such markets.

Many models of producer competition often operate under the assumption of commonly known capacities — an assumption that is difficult to motivate given the connection among capacities, wholesale prices, and overall market allocations. In particular, just as retailers may have an incentive to manipulate their orders from capacity-constrained producers [6, 15, 20], producers may have an incentive to obfuscate their output capacity in order to obtain more favorable terms from retailers.

The subject of private producer information and its impact on competition in supply chains has recently been considered in various papers. Some researchers have studied a situation in which producers, who have private capacity information, offer menus of quantity-price contracts to a profit-maximizing retailer [16, 24]. Others, including Albeniz and Talluri [22], consider a dynamic model of price competition between producers with limited capacity that must be sold before a deadline; they argue that "firms' best-response functions, however, are based on full information of the purchases [by customers] and capacities [of producers]. Although the concept of equilibrium itself is independent of such observabilities, it calls into question whether the two firms reacting to each other's strategy will ever be able to reach such equilibrium." They then go on to argue that the information, with known valuations, is coded in the price paths. We add to this literature by studying instances where producers strategically choose wholesale prices — knowing they alter

[1] `www.theglobeandmail.com/report-on-business/economy/the-global-shipping-news/article1744355`

[2] Papers in this literature are primarily set in the context of repeated interactions, e.g., repeated auctions.

perceptions about their privately known capacities — in order to attain higher profits.

While the literature has also studied the impact strategic[3] shortages may have on consumer behavior [31, 32], it has failed to properly address the impact strategic shortages (or the appearance of shortages) may have on competing producers. With this paper, we seek to fill in this gap in the literature. Ye et al. [40] independently study the effects of obfuscation in a framework where producers compete in *quantity* rather than in price. They show that high-capacity firms may signal their capacity through overproduction as a means to decrease the intensity of future competition. Our paper complements theirs by studying the analogous framework where producers compete in price. In contrast to their findings, we show that producers may choose to underproduce in order to obfuscate their capacities and appear weaker to competitors, dampening the competitive landscape and raising wholesale prices.

Our base model consists of two stages. In the first stage, producer j privately learns his capacity, and subsequently chooses his level of production. Producer i does not observe producer j's capacity directly, but its prior distribution is common knowledge. In each of the two stages, each of the producers simultaneously offers a wholesale price to the retailer, which in turn sets retail prices. Retail prices set in the first stage are common knowledge in the second stage. Hence, producer i is able to infer j's production level in the first stage, and may be able to glean information about j's underlying capacity constraint.

In the second stage, obfuscation may have already taken place: with some probability (possibly 0), producer i believes j's capacity to be different from j's true capacity. We find the Nash equilibrium of this stage, and examine the impact that capacity obfuscation can have on equilibrium prices and profits in both stages. Folding the game back to its first stage, we show that obfuscation can allow producers to tacitly collude and charge higher wholesale prices, thus moderating competition among producers.[4] The intuition for this result is the following: if producer i perceives j's capacity to be lower, he will price less aggressively (i.e., set a higher wholesale price), which in turn allows producer j to price less aggressively. We find that in equilibrium, both producers' profits increase, retail and wholesale prices increase, but the retailer's profit decreases.

The first stage of the game also allows us to determine the cost of obfuscation and characterize when a producer would choose to pursue such a strategy. In order to obfuscate, producer j must strategically cut his first-stage production, which may reduce his short-term profits, but results in profit gains in the second stage. We show that obfuscation is limited by its cost and by a strategic retailer's incentive to verify or reveal capacities (in effect, attempt to call a producer's "bluff"), where overall channel profits and consumer surplus may increase if the retailer can credibly counter the producers' incentives to obfuscate.

1.1 Related Literature

Our work falls into the broad area of joint production and pricing problems. For some recent reviews, see [7, 13, 37, 38, 39]. Porteus et al. [27] study a model with heterogeneous consumers

[3]There also exist uncertainties in the production process that are beyond producers' ability to control, which are widely studied in the literature; [8], for instance, studies the impact of such uncertainties in the market for influenza vaccines; [34] studies a retailer's optimal sourcing and inventory decisions when producers are exogenously unreliable. Our focus in this paper is on production uncertainties that arise intentionally.

[4]This result is reminiscent (but distinct) of [36], who show that a producer and a retailer (a power plant and a distributor in their framework) can both benefit from the producer misreporting capacity shortages (power outages at the plant). In the case of our model, in contrast, a producer benefits and the retailer loses from a competing producer misreporting his capacity.

and two capacitated firms producing differentiated products. The firms engage in sequential price competition. They give conditions under which the first mover can take advantage of its limited capacity by pricing relatively low, purposefully creating shortages to avoid direct competition with the follower. They show that such practices can lead to diminished competition among producers, which bears a similar flavor to one of our results (although ours is obtained in a simultaneous price-setting game with incomplete information[5]). In contrast to their findings, we show that as a result of obfuscation, both producers end up setting higher wholesale prices.

Özer et al. [26] study the problem of a producer soliciting private forecast information from a manufacturer, where the manufacturer has an incentive to exaggerate its demand forecast. They demonstrate that while theory predicts a "babbling" or cheap-talk equilibrium [11], where the manufacturer's report is uninformative and is subsequently ignored by the producer, human-subject experiments can result in contradictory findings. In particular, the outcome is not cheap-talk — the manufacturer's report contains truthful information, the producer exhibits trust, and the efficiency of the supply chain is significantly higher than predicted. Their study motivates our model in the sense that although there is an inherent cheap-talk problem when producers share information about their capacities, such communication is not entirely "babbling," leaving room for obfuscation to occur. Moreover, we show that there are cases where even an obfuscated producer benefits from the pervasiveness of obfuscation, thus creating a pathway for sustainable collusion without trust. In contrast to [26], however, we find that the overall efficiency of the supply chain is reduced due to diminished competition.

Our framework is also different from direct communication and associated cheap-talk issues recently explored in [1] and [9]. This is because our model consists of indirect communication (or rather, signaling) as the result of strategically downsizing output. The "primary" action of a producer in our framework is to set a wholesale price, and this action has a spillover signaling effect, which the producer can use strategically. In the aforementioned works, an agent's actions can consist of directly communicating relevant parameters, such as demand or inventory forecasts, which leads to considerably different strategic considerations — whether the communication contains actionable information or not, for instance. At the same time, our results are complementary to these works. We show that strategically cutting output can increase the amount of informational noise in a supply chain, but a costly counter-action by a strategic retailer can induce a producer to behave truthfully, reducing noise. Complementary to [9], we also show that tacit collusion can in fact reduce supply-chain efficiency. We further show that collusive and truthful behaviors can be sustained as part of an equilibrium, without incorporating behavioral components such as trust. Along with differences in terms of the vehicle of information transmission, our model also examines a different competitive landscape. In contrast to our work, [9] consider the role of informal communication about demand forecast information between a retailer and his manufacturer, and [1] study the problem of information transmission in a retail operations setting, focusing on strategic communication between a retailer and its customers about inventory availability. Our model, on the other hand, focuses on the impact of capacity obfuscation on channel competition in a channel structure with two competing producers and one intermediary that retails both producers' products.

Our model thus nests in the signaling framework described in [30] (see [29] for a survey). Closest to our work, Cachon and Lariviere [6] also explore the use of a producer's wholesale price (and

[5] A simultaneous price-setting game is the appropriate informational framework in our context, as producers are not informed of each other's wholesale prices when they set their own.

corresponding output sold) as a signaling device. In a similar fashion, information in our model is communicated noisily in equilibrium because the high-type producer is able to set a wholesale price and sell output levels that correspond to a low-type producer. Cachon and Lariviere consider a supply chain in which a single producer sells to several downstream retailers. The producer has limited capacity, and retailers are privately informed of their optimal stocking levels. They show that a broad class of mechanisms with constant wholesale prices are prone to manipulation: retailers will order more than they need to gain a more favorable allocation. Furthermore, a manipulable mechanism may lead the producer to choose a higher level of capacity than she would under a truth-inducing mechanism, though switching to a truth-inducing mechanism can lower profits for the producer, the supply chain, and retailers. In contrast to [6], we relax the assumption on constant wholesale prices and study competition among producers rather than among retailers. We find that due to such competition, a manipulating producer will actually choose to understate his capacity in order to mitigate an aggressive pricing strategy by a competitor. Hence, we find that competition among producers mitigates the retailer manipulation effect characterized in [6], and so our work is complementary to theirs. Also, in line with their work, we find that producers may actually benefit from obfuscation.

Gümüs et al. [16] present a related model where a producer may face capacity uncertainty, which impacts other market actors' beliefs about his capacity. They show that the producer can alleviate some of the market uncertainty by signaling the extent of his capacity volatility through guaranteed-output contracts. In contrast, a high-capacity producer in our framework works to *facilitate* market uncertainty by appearing to have a tighter capacity constraint. Relatedly, [25] and [18] examine how a manufacturer signals an imperfect forecast of demand by the contract offered to a producer and by the timing and level of capacity investments. Other related works study models where signals are conveyed through production decisions, with actors similarly possessing private information about demand forecasts [10, 12, 21]. Although a producer in our model possesses private information about his own production capacity rather than joint demand parameters (and there is no demand uncertainty), the producer's individual capacity interacts with market pricing, and thus ends up affecting other market actors. Moreover, a producer in our framework may seek to increase rather than alleviate market uncertainty. In line with this observation, Anand and Goyal [2] study a model of horizontal competition between an informed and uninformed firm with a common upstream producer. They show that an informed firm's incentive to control information leakage in the supply chain can alter its behavior, underscoring the importance of taking into account such strategic concerns.[6]

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium of the second stage of the game, whereas Section 4 folds the game back to its initial stage to show when a producer would choose to obfuscate his capacity. Section 5 extends the model to consider a strategic retailer who may choose to deter capacity obfuscation. Concluding remarks are presented in Section 6.

[6]On the consumer side, [31] show that strategic shortages can be used by producers as a quality signaling device. In particular, they show that high quality producers may strategically create scarcity for their products in order to signal consumers about their product quality; [32] relatedly shows that producers may choose to cut capacity in the presence of speculators. These works are complementary to ours. They show that producers have incentives to cut capacity when doing so impacts consumer behavior. Our model, on the other hand, studies a producer's incentive to cut capacity when doing so impacts a *competing producer's* behavior.

2 Model

The model consists of a retailer that is selling two competing product brands, i and j, each provided by a different producer, also identified by i and j, respectively. There are two stages, where in each stage competition among producers is modeled as a simultaneous price-setting game. To emphasize the relative capacity limitation of producer j, we assume that his competitor, producer i, is capacity *un*constrained, whereas producer j may be capacity constrained.[7]

At the beginning of the first stage, producer j privately learns his capacity, which can be either low (denoted by K_j^L), or high (denoted by K_j^H). For technical simplicity, we assume that the two stages occur within close time proximity, whereby capacity constraints are constant across the two stages.[8] Producer i's prior beliefs about j's capacity are as follows: producer i believes j's capacity to be K_j^L (K_j^H) with probability θ $(1-\theta)$. All of the elements of the game except producer j's capacity are assumed common knowledge. Both producers discount the future according to a common discount factor $\delta \in [0,1]$. The solution concept we employ is Perfect Bayesian Equilibrium.

In each of the two stages, each of the producers simultaneously offers a wholesale price $w_{g,t}$, where $g \in \{i,j\}$ and $t \in \{1,2\}$, to the retailer, which in turn controls market demand by setting prices for each stage, given by $(D_{i,t}, p_{i,t})$ and $(D_{j,t}, p_{j,t})$. Retail prices set in the first stage become common knowledge in the second. This is easily motivated, as any party should be able to observe the prices set by the retailer. Thus, at the end of the first stage, given $p_{j,1}$, producer i can infer $D_{j,1}$ from the retailer's problem.[9] If producer j's sales quantity in the first stage, $D_{j,1}$, exceeds K_j^L, producer i knows for certain that j's capacity is K_j^H. If, on the other hand, $D_{j,1} \leq K_j^L$, producer i does not immediately learn j's capacity. This is because producer j may strategically choose to underproduce in order to conceal having a high capacity. Let the function $\theta_2(D_{j,1})$ denote producer i's posterior belief regarding producer j having a low capacity in the second stage, given sales quantity $D_{j,1}$. (For notational simplicity, we henceforth refer to $\theta_2(D_{j,1})$ as θ_2.)

Let $\Pi_{j,1}^H(K,\theta)$ and $\Pi_{j,1}^L(K,\theta)$ denote producer j's profits in the first stage when he produces K units and his capacity is K_j^H (High) and K_j^L (Low), respectively, and producer i's prior belief is θ. Similarly, we denote producer j's profits in the second stage, given producer i's posterior belief θ_2, as $\Pi_{j,2}^H(K,\theta_2)$ and $\Pi_{j,2}^L(K,\theta_2)$.

We assume that the retailer seeks to maximize category profits. The decision variables for each producer are the wholesale prices to be offered to the retailer, $w_{i,t}$ and $w_{j,t}$, for producers i and j, respectively, while the retailer sets corresponding retail prices $p_{i,t}$ and $p_{j,t}$ for their products.
We assume market demands (corresponding to sales quantities) for the two products have the form:

$$D_{i,t}\left(p_{i,t}, p_{j,t}\right) = \alpha - p_{i,t} + \gamma p_{j,t} \qquad \text{and} \qquad D_{j,t}\left(p_{i,t}, p_{j,t}\right) = \alpha - p_{j,t} + \gamma p_{i,t}$$

where $\alpha > 0$ and $\gamma < 1$ are parameters of the demand function representing market size and the degree of product substitutability between the two products, respectively. Linear demand functions of the form described above are commonly used in the literature. Although it is possible to incorporate nonlinear demand functions, there are no major qualitative benefits to doing so,

[7] Studying the case where producer i is unconstrained allows us to keep the model relatively simple by eliminating some corner solutions, as well as shed light on the dynamics resulting from a single obfuscating party. The case where both producers obfuscate lends similar qualitative results.

[8] The results go through provided there is sufficient correlation between capacities across stages.

[9] A strategic retailer is considered in Section 5, where we show that in the absence of commitment power by the retailer, our findings are robust to this extension.

and with the inclusion of capacity constraints, the problem is already quite complex. We do not require specific relationships between the variable costs of the two products or between $w_{i,t}$ and $w_{j,t}$. Similarly, we make no assumptions regarding the relationship between $p_{i,t}$ and $p_{j,t}$.

We first consider the retailer's decisions given the wholesale prices offered by producers i and j. In each stage $t \in \{1, 2\}$, the retailer's problem is to select prices $p_{i,t}$ and $p_{j,t}$ to maximize his total profits from the sale of both products:

$$\max_{p_{i,t}, p_{j,t}} \Pi_R = (p_{i,t} - w_{i,t}) [\alpha - p_{i,t} + \gamma p_{j,t}]^+ + (p_{j,t} - w_{j,t}) [\alpha - p_{j,t} + \gamma p_{i,t}]^+ \tag{1}$$

At this point, we assume that the retailer myopically sets prices in each stage; Section 5 extends the model to incorporate the possibility of a strategic retailer. Retail prices are constrained to be larger than the wholesale prices submitted by the two producers ($w_{i,t}$ and $w_{j,t}$) in order for the retailer to make a positive profit, but will not be so high as to make the sales quantities non-positive. Hence, we omit the 'positive part' in the expression representing demands/sales quantities in the exposition of the analysis. We also note that in contrast to [6], producer j sets a wholesale price to ensure that the retailer's ordered quantity does not exceed his capacity.

The objective in (1) is jointly concave in $p_{i,t}$ and $p_{j,t}$ (proof is standard and omitted), giving the profit-maximizing retail prices:

$$p^*_{i,t} = \frac{\alpha}{2(1-\gamma)} + 0.5 w_{i,t} \quad \text{and} \quad p^*_{j,t} = \frac{\alpha}{2(1-\gamma)} + 0.5 w_{j,t} \qquad t \in \{1, 2\} \tag{2}$$

3 Competition in the Second Stage

In this section, we characterize the equilibrium strategies in the second stage of the game. Let us assume that producer j has undertaken some action that, from the perspective of his competitor, makes it unclear whether his true production capacity is K_j^L or K_j^H. In particular, consider the case where producer j produced K_j^L in the first stage. From the perspective of his competitor, it is not clear whether j's capacity is in fact K_j^L or whether production in the first stage was kept low in order to obfuscate j's true capacity K_j^H. As a consequence, producer i forms beliefs over j's capacity. Recall that producer i's posterior beliefs about j's capacity are denoted by θ_2;[10] that is, producer i believes that j's capacity is K_j^L with probability θ_2 and K_j^H with probability $1 - \theta_2$.

3.1 Producer i's Problem

Let $w_{j,2}^L$ and $w_{j,2}^H$ denote second-stage wholesale prices that correspond to a low-capacity and high-capacity producer j, respectively. Given producer i's beliefs about j's capacity, as specified by θ_2, producer i's objective is to choose $w_{i,2}$ to maximize his expected profit:

$$\begin{aligned} \max_{w_{i,2}} \Pi(w_{i,2}, \theta_2) = \; & \theta_2 \left\{ (w_{i,2} - c_i) \left[\alpha - \left(\frac{\alpha}{2(1-\gamma)} + 0.5 w_{i,2} \right) + \gamma \left(\frac{\alpha}{2(1-\gamma)} + 0.5 w_{j,2}^L \right) \right] \right\} & (3) \\ & + (1 - \theta_2) \left\{ (w_{i,2} - c_i) \left[\alpha - \left(\frac{\alpha}{2(1-\gamma)} + 0.5 w_{i,2} \right) + \gamma \left(\frac{\alpha}{2(1-\gamma)} + 0.5 w_{j,2}^H \right) \right] \right\} \\ \text{subject to:} \quad & w_{i,2} \geq c_i & (4) \end{aligned}$$

[10] Prior beliefs (at the beginning of the first stage) are specified by θ, whereas θ_2 denotes posterior beliefs at the beginning of the second stage.

The first (second) term represents producer i's profit conditional on facing a producer j with capacity type K_j^L (K_j^H), weighed by producer i's posterior belief that j has capacity K_j^L (K_j^H). Taking the derivative of Equation (3) with respect to $w_{i,2}$, and setting it equal to zero, we obtain producer i's reaction function for optimally setting his wholesale price, $w_{i,2}\left(w_{j,2}^H, w_{j,2}^L\right)$, given by:

$$w_{i,2}\left(w_{j,2}^H, w_{j,2}^L\right) = \frac{1}{2}c_i + \frac{\alpha + \gamma w_{j,2}^H + \theta_2 \gamma \left(w_{j,2}^L - w_{j,2}^H\right)}{2} \tag{5}$$

3.2 Producer j's Problem

Knowing the retailer's pricing strategy, and anticipating producer i's wholesale pricing strategy, producer j solves two profit maximization problems, one to find $w_{j,2}^L$, and a second to find $w_{j,2}^H$. We show the formulation for the former.

$$\begin{aligned} \max_{w_{j,2}^L} \Pi_{j,2} \quad &= \quad (w_{j,2}^L - c_j)\left[\alpha - p_{j,2}\left(w_{i,2}, w_{j,2}^L\right) + \gamma p_{i,2}\left(w_{i,2}, w_{j,2}^L\right)\right] \\ s.t. \quad & \quad w_{j,2}^L \geq c_j \\ & \quad \alpha - p_{j,2}\left(w_{i,2}, w_{j,2}^L\right) + \gamma p_{i,2}\left(w_{i,2}, w_{j,2}^L\right) \leq K_j^L \end{aligned}$$

We note that the problems faced by producer j have the same structure as that in the complete information benchmark (shown in Appendix A).

3.3 Second-Stage Equilibrium

Based on the specification of the model's parameters, including K_j^H and K_j^L, three capacity configurations emerge in the second-stage equilibrium, denoted by UU, UC, and CC.[11] The (U, U) two-tuple represents the case where producer j's capacity is not binding at level K_j^H nor at level K_j^L; whereas the (U, C) two-tuple represents the case where, in equilibrium, producer j's capacity is not binding at level K_j^H but is binding at level K_j^L; finally, the (C, C) two-tuple represents the case where producer j's capacity is binding at both K_j^H and K_j^L in equilibrium.

Using the wholesale price reaction functions for producers i and j (as specified in Section 3.1 and in Appendix A), we can solve for the second-stage equilibrium wholesale prices, sales quantities, and profits when producer j's capacity is given by K_j^H and K_j^L, leading to the following result.

Lemma 1 *Producer j does not benefit from appearing to have a capacity level that is higher than his true capacity; that is, a type K_j^H producer may have an incentive to appear to be a type K_j^L, whereas a type K_j^L has no incentive to appear to be a type K_j^H.*

The proof of Lemma 1 and all equilibrium (prices, sales quantities, and profits) results are in Appendix B. The intuition for why a higher perceived capacity can hurt producer j is that it may lead producer i to price more aggressively (that is, set a lower wholesale price), and this in turn can force producer j to sell a similar amount (e.g., at most his capacity, K_j^L) at a lower price.

Equilibrium Capacity Configurations in the $\left(K_j^H, K_j^L\right)$-space:
To summarize our findings thus far, Figure 1 depicts the resulting second-stage equilibrium implied by different (K_j^H, K_j^L) pairs, along with three important capacity threshold lines (i.e., $K_j^L = K_j^H$, $D_{j,2}^{UU^*}$, and $D_{j,2}^{H,UC^*}$), where $D_{j,2}^{UU^*} \equiv D_{j,2}^{L,UU^*} = D_{j,2}^{H,UU^*}$ represents producer j's sales quantity

[11] Appendix B details the derivation of each capacity configuration.

when he is unconstrained in equilibrium whether he is of type K_j^H or K_j^L, and $D_{j,2}^{H,UC^*}$ represents producer j's sales quantity when he is not constrained in equilibrium as a type K_j^H, but would be constrained as a type K_j^L. Observe that to the southeast of the 45-degree line representing $K_j^L = K_j^H$, we have the region specified by Lemma 1, where $K_j^L \leq K_j^H$. For $K_j^H \geq K_j^L \geq D_{j,2}^{UU^*}$, the resulting second-stage equilibrium exhibits the (U,U) configuration; that is, both a low-type and a high-type producer j would be unconstrained in equilibrium. For $K_j^H \geq D_{j,2}^{H,UC^*}$ and $K_j^L \leq D_{j,2}^{UU^*}$, the (U,C) configuration is obtained. For $K_j^L \leq K_j^H \leq D_{j,2}^{H,UC^*}$, the (C,C) configuration is obtained; that is, both a high-type and a low-type producer j with capacity levels K_j^H and K_j^L, respectively, will be constrained in equilibrium.

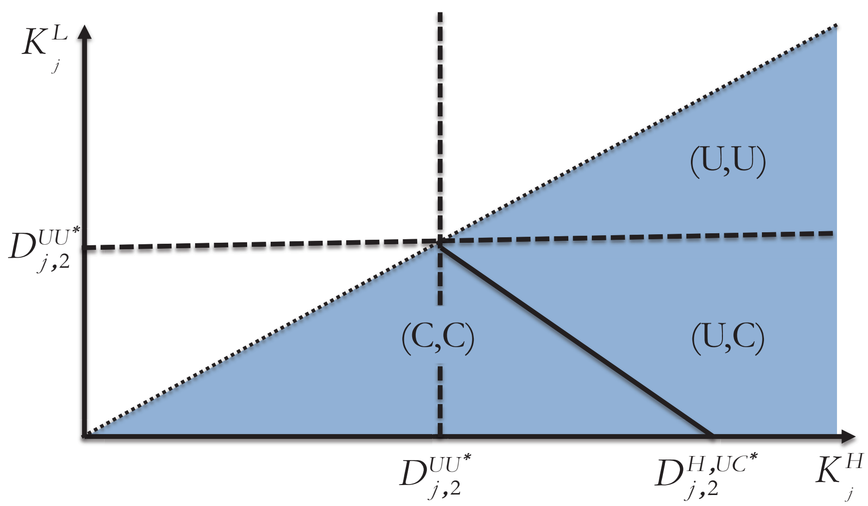

Figure 1: Equilibrium Capacity Configurations in the (K_j^H, K_j^L)-space.

3.4 Profits in the Second Stage

In this subsection, we first show that a high-capacity producer j has an incentive to manipulate perceptions about his underlying capacity due to gains in second-stage profits. We then discuss how obfuscation affects the profits of the obfuscated producer and the retailer.

Proposition 1 *In the second stage, under both the (U,C) and (C,C) configurations, given $K_j^L < \min\{K_j^H, D_j^{UU^*}\}$, a high-capacity producer j strictly benefits from obfuscating i's beliefs.*

Proposition 1 suggests that predictions generated from models that ignore the possibility of obfuscation may be distorted, in the sense that these predictions ignore a possibility that is relevant to determining equilibrium prices and profits. In effect, by pretending to be more constrained, producer j is able to lead producer i to price less aggressively (i.e., set a higher wholesale price), which in turn allows producer j to also price less aggressively. The result is a less competitive landscape, where producer j secures higher profits.[12]

When it comes to producer i's profits, things are not as clear cut. In particular, due to obfuscation, producer i experiences two effects. The first effect, that of *moderated competition*, is clear:

[12]The reverse perception, i.e., that of producer j being less constrained than he truly is, will only intensify price competition, leading to lower profits for a low-capacity producer j.

since producer i prices less aggressively, so will producer j (as specified by his wholesale price reaction function, shown in Table 2 in Appendix B). In other words, both producers will increase their wholesale prices, which can work in i's favor. The second effect, on the other hand, is that of *substitution*. Because producer j is strategically responding to i's wholesale price, he trades off a higher price per unit with a greater number of total units sold. In equilibrium, producer j equates the marginal returns from each, which results in the retailer partially substituting away from i's product to j's, thus negatively impacting i's profit. In net, changes in producer i's profits can be positive or negative.

It is instructive to consider how producer i's beliefs regarding j's capacity affect profits. That is, do stronger beliefs regarding j's (potentially understated) apparent capacity improve or detract from producer i's profit? Such a consideration is insightful because if producer i benefits from stronger beliefs, it would suggest that producer i's incentives to possibly take actions to alleviate j's obfuscation (by, for instance, verifying j's capacity) are diminished. The following result characterizes cases where i benefits from stronger beliefs.[13]

Proposition 2 *Producer i's second-stage profit is increasing in θ_2 if for $mn \in \{UC, CC\}$, $K_j^L \in [\underline{K}^{mn}, \min\{K_j^H, D_{j,2}^{UU*}\}]$, where capacity thresholds are specified by $\underline{K}^{UC} = (\alpha - c_j + \gamma c_i)/4$ and by $\underline{K}^{CC} = -\left[(1+\gamma)\gamma\alpha - (1-\gamma^2)\gamma c_i - (4-2\gamma^2)K_j^H\right]/4\left[1-\gamma^2\right]$.*

A direct implication of Proposition 2 is that when K_j^L is sufficiently high, the moderated competition effect dominates, and i's second-stage profits increase; that is, i benefits from j's obfuscation. Figure 2 shows the impact of θ_2 on producer i's profits in two different examples.

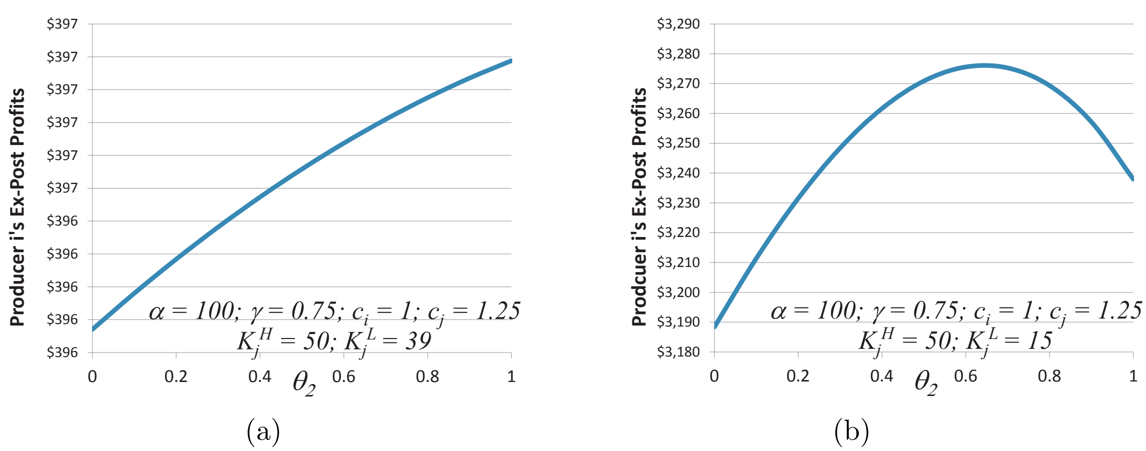

Figure 2: Second-stage profits for producer i under the (U, C) configuration.

In the left pane (Figure 2(a)), the professed capacity is not significantly constraining (i.e., K_j^L is relatively high), and so the moderated competition effect dominates. In the right pane (Figure 2(b)), while the products are highly substitutable, the professed capacity, K_j^L, is exceedingly low. Initially,

[13]We emphasize that the proceeding results regarding profits increasing in θ_2 directly imply that the party to which the profits belong is better off under obfuscation. Of course, one could have stated that, for instance, "Producer i's profit is higher under obfuscation," but our results are in fact stronger, as they also provide insight regarding the difference between the party's profit with and without obfuscation.

for lower values of θ_2, the moderated competition effect dominates that of substitution, but this relationship reverses for higher values of θ_2, as producer j takes advantage of i's perception regarding his obfuscated capacity to take away some of i's sales.

While the net effect on producer i's profit may be ambiguous, the effect of obfuscation on the total profits of producers is clear cut, as the following proposition indicates.

Proposition 3 *Under both the (U, C) and (C, C) configurations, the sum of producers' second-stage profits is increasing in θ_2.*

Proposition 3 is the culmination of a series of results regarding producers' profits, showing that uncertainty regarding capacity constraints can indeed benefit the producers in the supply chain. Producers' profits increase in the intensity parameter, θ_2, as competition is further moderated. In effect, a higher θ_2 brings the second-stage equilibrium closer to an outcome where the production of both brands is horizontally integrated (although such an outcome is never actually reached), and where obfuscation as a form of tacit collusion has a greater likelihood of being sustained. Moreover, while there are cases where producer i does not benefit from j's obfuscation, the increase in j's profit more than offsets any loss producer i may incur.

We next examine the impact of obfuscation on consumer surplus, on the retailer's profit, and on channel profits in the second stage.

3.5 Impact on Consumer Surplus, Retailer Profits, and Channel Profits

The effect obfuscation has on prices, both retail and wholesale, as the following result shows, is harmful to both consumers and the retailer.

Proposition 4 *Under both the (U, C) and (C, C) configurations, retail and wholesale prices ($p_{g,2}$, $w_{g,2}$), $g \in \{i, j\}$, are increasing in θ_2, while the retailer's second-stage profits are decreasing. Second-stage channel profits are decreasing in θ_2 under the (C, C) configuration; under the (U, C) configuration, channel profits are decreasing in θ_2 when $\alpha \geq \frac{1}{1-\gamma^2} c_i$.*[14]

While the sum of producers' profits increases as a result of obfuscation (more strongly, under the conditions of Proposition 2, individual producer profits increase as well), as evident from the above result, consumers are strictly worse off. In other words, obfuscation allows producers to collude without ever formally agreeing to do so. Said another way, obfuscation enables producers to coordinate on a more collusive outcome that dampens some of the effects of competition. In contrast to the producers (but in line with consumers), the retailer's profits are always decreasing in θ_2. The retailer's second-stage profit functions under the different equilibrium capacity configurations are detailed in the Appendix, along with a proof of the Proposition.

From Proposition 4, channel profits are always decreasing in the intensity of obfuscation under the (C, C) configuration. For the (U, C) configuration, the parameter restriction for decreasing channel profits is easily understood. Since $\alpha \geq c_i$, the restriction requires that either the market is large and/or the products are sufficiently differentiated. We emphasize that this is a sufficient condition, and note that the necessary condition, which is provided in the Appendix, is significantly

[14] A stronger sufficient condition for second-stage channel profits to be decreasing in θ_2 is given by $\alpha \geq \frac{4-3\gamma^2+\gamma^4}{4-3\gamma^2-\gamma^3} c_i$.

weaker. We also provide in the Appendix a knife-edge parameter specification where channel profits are actually increasing in the intensity of obfuscation under the (U, C) configuration.

Since obfuscation leads to an outcome that resembles one in which firms collude, it is intuitive that while the sum of producers' profits subsequently increases, consumer surplus, retailer profit, and channel profits are diminished.[15]

Next, we fold the game back into its first stage, where the cost of obfuscation is determined endogenously.

4 Competition in the First Stage

Up to this point, we have studied the second stage of the game, where producer j, the obfuscator, had already influenced producer i's beliefs about his capacity. These beliefs were taken as a parameter that resulted from some past actions. We now show that obfuscation can arise as part of actions taken in a previous stage.

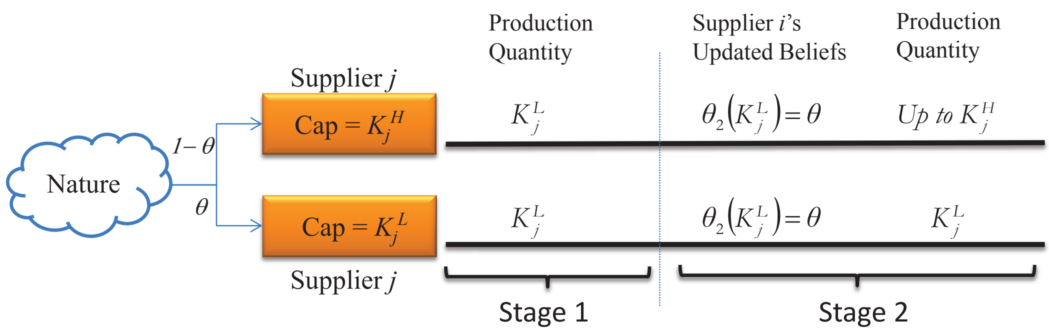

Figure 3: Strategic Capacity Cutting Equilibrium

As illustrated in Figure 3, consider an equilibrium of the game where both high and low capacity types (K_j^H and K_j^L) of producer j produce K_j^L in the first stage. In other words, producer i is unable to glean any information from the first-stage retail price $p_{j,1}$. This is because $p_{j,1}$ conveys no new information about j's type. Thus, producer i's posterior belief is the same as his prior; that is, $\theta_2(K_j^L) = \theta$. Therefore, the second stage subgame when producer j's true capacity is K_j^H, given that $\theta_2(K_j^L) = \theta$, is identical to the second-stage game analyzed in Section 3.

Let $\Pi_{j,1}^H(K, \theta)$ and $\Pi_{j,1}^L(K, \theta)$ denote producer j's profits in the first stage when he produces K units and his capacity is K_j^H and K_j^L, respectively, and producer i's prior belief is θ. In the proceeding analysis, we focus on the (C, C) configuration, i.e., where both high and low types of producer j face a tight capacity constraint; the (U, C) case is similar.

Let us assume that as part of the first-stage equilibrium, both types of producer j behave as type K_j^L. Producer i then anticipates this strategic pooling by producer j's types. Hence, producer i realizes that regardless of producer j's true capacity, j will produce at K_j^L in the first stage. Then, if producer j in fact has a high capacity K_j^H, it is a best response for j to underproduce at K_j^L in

[15]This finding is reminiscent of [17] in the sense that once asymmetric information is introduced, the payoff of the party with the private information is improved while the other party's payoff is diminished. However, there is a third party in our model in the form of a competing producer, and, as we show, the impact on its payoff is ambiguous.

the first stage when

$$-\left(\Pi^H_{j,1}(K^H_j,1)-\Pi^H_{j,1}(K^L_j,1)\right)+\delta\left(\Pi^H_{j,2}(K^H_j,\theta)-\Pi^H_{j,2}(K^H_j,0)\right)\geq 0, \tag{6}$$

that is, if a high-type producer j's potential loss in first-stage profit due to underproducing is exceeded by his discounted gain in second-stage profit. From the perspective of a low-type producer j, producing at full capacity is still the optimal choice in the first stage, since producer i now anticipates production at K^L_j with certainty.[16] Moreover, a low-type producer j benefits from the high-type's obfuscation due to less intense competition in the first stage (and would be indifferent when $\delta = 1$). Solving the inequality, we have the following result.

Proposition 5 *There exists a threshold $\underline{\delta} \in (0, 1)$ such that given any discount factor $\delta \in [\underline{\delta}, 1]$, a high-type producer j obfuscates by underproducing in the first stage.*

The expression for $\underline{\delta}$ and the proof along with the complete equilibrium specification are in Appendix B.6. Whether $\underline{\delta} \in [0, 1]$ depends on the parameter specification. When $\underline{\delta} < 0$, obfuscation always takes place — these are cases where producer j benefits directly from obfuscating in the first period. Intuitively, these cases are characterized by K^H_j being relatively high, which leads to a Prisoner's Dilemma type of situation between producers where both i and a high-type j set low prices and "overproduce"; pursuing obfuscation enables the producers to coordinate on an outcome with less intense competition (i.e., higher wholesale prices). When $\underline{\delta} > 1$, obfuscation never pays off. We further have that $\underline{\delta}$ is decreasing in c_j, K^H_j, K^L_j, and θ, and is increasing in α and c_i.

From Proposition 5, given a discount factor δ that is sufficiently high, a producer j whose realized capacity is high benefits from strategically cutting production in the first stage. Intuitively, producers who put sufficient weight on future profits may be willing to incur losses in the present in order to facilitate future uncertainty about their true capacity. Moreover, as these present losses diminish, this cost-benefit analysis tilts further in favor of obfuscation; that is, $\underline{\delta}$ is lower.

As the following result shows, a producer j whose realized capacity is low, as well as the competing producer i, both benefit from the high type's strategy (and their benefit holds more broadly, for any discount factor δ).

Proposition 6 *The expected present-discounted total profits of a low-capacity producer j and of producer i are higher when a high-capacity producer j underproduces in the first stage.*

Figure 4 provides an illustration of present-discounted expected (total) profits for the producers, conditional on producer j's type, for the cases with and without obfuscation. The top two illustrations depict producers j and i's profits with and without obfuscation, conditional on producer j being a high type, for an interior $\underline{\delta} \in (0, 1)$. The bottom two illustrations depict the producers' profits conditional on a low-type producer j. As the figures demonstrate, producer i and a low-type producer j always benefit from the prevalence of obfuscation — due to less intense competition. A high-type producer j's overall preference depends on the underlying parameters.

Propositions 5 and 6 illustrate that even when there are earlier costs associated with obfuscating a competitor's posterior beliefs, the incentive to obfuscate can exist.

To provide some intuition, consider a high-capacity producer j who chooses to cut production in the first stage. While producer j's gain from underproducing is realized in the second stage,

[16] An appropriate off-equilibrium belief specification for producer i is given by $\theta_2(K) = \theta$ for all $K \leq K^L_j$.

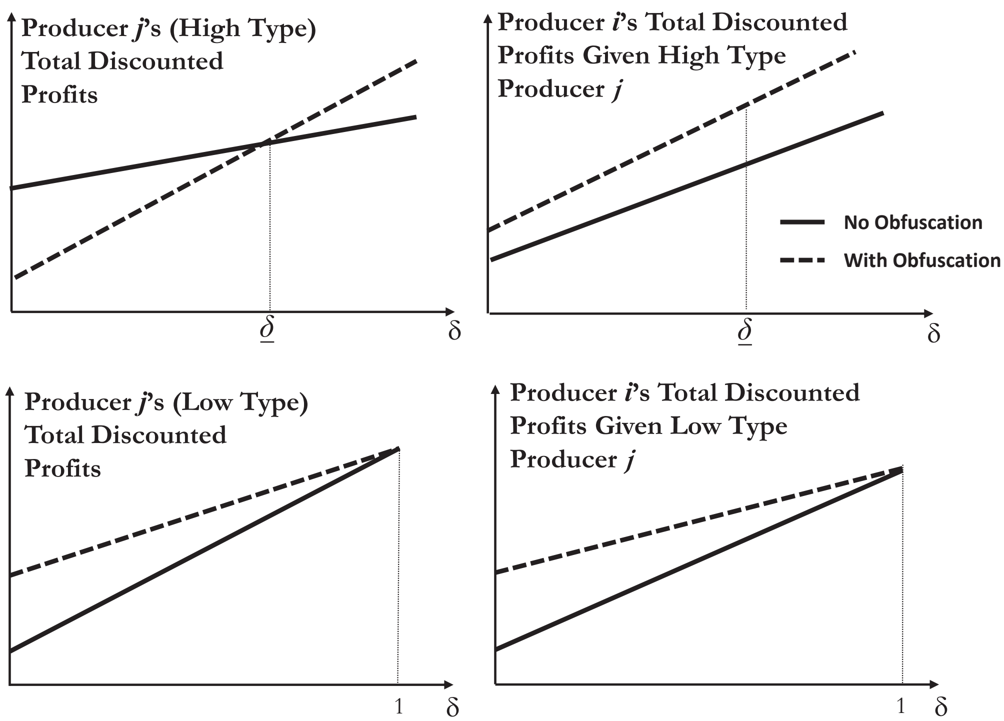

Figure 4: An illustration of producers' present-discounted expected profits conditional on j's type as a function of δ.

producer i's immediate benefit is twofold. First, given j's choice of producing less and setting a higher wholesale price, the retailer demands a higher amount of i's product (a 'substitution' effect). Second, since producer i recognizes j's strategy in equilibrium, he can account for j's higher wholesale price in the first stage and raise his own wholesale price (a 'moderated competition' effect). From the perspective of a low-capacity producer j, the result of the high type's pooling behavior in the first stage delivers certainty to the competing producer i that j's wholesale price will be high, which enables both producers to raise prices and benefits the low-type producer j (and this benefit exceeds any loss the low type incurs in the second stage as a result of obfuscation). Our next result shows the overall impact on the retailer's profit, on overall channel profits, and on consumer surplus.

Proposition 7 *The retailer's present-discounted expected total profit and consumer surplus are lower as a result of producer j obfuscating his capacity. Furthermore, there exists a threshold $\bar{K}_j^L > 0$, such that for all $K_j^L \leq \bar{K}_j^L$ channel profits decrease as a result of obfuscation.*

It follows that the benefit to both producers resulting from producer j cutting production in the first stage comes at a cost for the other parties in the supply chain. In particular, as wholesale prices increase, not only does producer j avail less capacity to the retailer, but the retailer's potential profit margins are diminished, lowering the retailer's profit. In response, the retailer raises his own prices, which lowers consumer surplus and the quantities that consumers demand.

5 A Strategic Retailer Countering Obfuscation

We now consider an extension of the model where the retailer is able to take actions to counter producer j's obfuscation. In the base model, in effect, we had been operating under the assumption that the retailer is not strategic. An alternative assumption is that the retailer cannot communicate any new information (such as j's wholesale prices) to producer i. This can be the case under any (combination) of the following circumstances: (i) because this information is supplied to the retailer late in the production process; (ii) because the retailer is contractually obligated not to disclose such information, i.e., there is a penalty for breaching a contract with producer j; or (iii) because the retailer is unable to credibly (let alone verifiably) disclose such information to producer i without some external certification.[17] Below we consider two approaches that a strategic retailer might take to alleviate obfuscation.[18]

5.1 Deterrence through Disclosure

Under the first approach, let us suppose that in the second stage, the retailer is able to credibly and verifiably disclose producer j's (true) capacity (or alternatively, producer j's wholesale price) to producer i, at some cost, denoted by c_V, prior to i's wholesale pricing decision. Thus, the cost c_V to the retailer represents either (i) the cost of obtaining information about producer j's (true) capacity early on; (ii) the cost of breaching the contract with producer j; (iii) the cost to certify the retailer's information; or (iv) any combination of these costs. For (i), it might be possible for the retailer to glean information regarding producer j's capacity in the first stage. For instance, the retailer could lower product j's first-stage retail price, $p_{j,1}$, and subsequently request to acquire more units from j. If j complies, his true capacity is revealed. Granted, doing so may come at a cost to the retailer due to lower revenues if producer j does not comply and producer i's wholesale price is unchanged.

For technical simplicity, let us assume that it is common knowledge that the retailer possesses complete information regarding producer j's capacity. Once the retailer credibly discloses j's capacity to producer i, producer i is no longer affected by producer j's obfuscation. In fact, producer i will respond optimally to producer j's true capacity according to its best-response pricing function. If it is indeed optimal for the retailer to disclose j's capacity to i in equilibrium, then producer j will foresee such a disclosure taking place, and will price accordingly. In effect, the retailer reverts the game to the complete-information benchmark (i.e., where producer j's capacity is known in the second stage). Hence, the retailer will choose to disclose producer j's capacity to i if the cost of doing so, c_V, is lower than the retailer's gain in profit from reverting to the complete-information outcome.

An upper bound on c_V is given by the retailer's profit gain in the second stage under the complete-information outcome relative to the outcome with obfuscation, as characterized by the following result.

Proposition 8 *There exist constants $\bar{c}_V^C, \bar{c}_V^U > 0$, such that if producer j is constrained (unconstrained) in the complete-information benchmark, the retailer's benefit from alleviating obfuscation*

[17] There is an inherent cheap-talk problem in this case [11], since the retailer would always prefer to disclose a lower wholesale price for producer j to i in order to intensify competition among the producers and induce i to set a lower wholesale prices.

[18] A related approach taken in the literature is for the retailer to pre-commit to a quantity acquisition prior to producers ramping up their production capacities [4, 14].

is bounded above by $\bar{c}_V^C$ ($\bar{c}_V^U$). Moreover, $\bar{c}_V^C$ and $\bar{c}_V^U$ are non-increasing in K_j^L; that is, the retailer's incentive to counter obfuscation diminishes as K_j^L increases.

The retailer's equilibrium profits under each of the capacity configurations and the respective upper bounds are detailed in the Appendix. It follows that if the retailer is able to credibly communicate information about producer j's capacity to producer i, he will choose to do so when the cost of disclosing the information, as denoted by c_V, is less than $\bar{c}_V^m$, where $m \in \{C, U\}$ indicates j's capacity configuration.

5.2 Deterrence through Output Reduction

Let us now consider the possibility that a strategic retailer may make it more costly for a high-capacity producer j to obfuscate by acquiring an inefficiently low quantity from j (from a per-period profit-maximizing perspective) in the first stage. More specifically, suppose that in the first stage, if the retailer is offered a wholesale price from producer j that indicates low capacity, the retailer chooses to acquire fewer units from j, thus increasing the opportunity cost j incurs due to underproducing. In other words, the retailer, not knowing j's true capacity in the first stage, gambles by making it costlier for a producer j that has a high capacity to pretend to have low capacity. This strategy can work for the retailer as long as the added cost imposed on an obfuscating producer j exceeds his benefit from obfuscation. Moreover, provided that this strategy works, it can benefit the retailer and be sustained in equilibrium when the retailer's gain from disincentivizing a high-capacity producer j from underproducing exceeds the retailer's potential loss from underselling j's product in the first stage when j's true capacity is low.

Under this approach, the retailer commits in advance to the following policy: If producer j sets a wholesale price that corresponds to a low capacity level, K_j^L, the retailer will only order $q^R < K_j^L$ units from producer j. For this approach to benefit the retailer, producer j must be deterred from obfuscating. Hence, it is necessary that the retailer sets a quantity q^R that satisfies the inequality $\Pi_{j,1}^H(K^*, \theta) - \Pi_{j,1}^H(q^R, \theta) \geq \delta(\Pi_{j,2}^H(K^*, \theta) - \Pi_{j,2}^H(K^*, 0))$, where K^* denotes the profit-maximizing production level when producer j's capacity is K_j^H. That is, the retailer must order a sufficiently low quantity such that a high-capacity producer j's opportunity cost of cutting first-stage production exceeds his second-stage benefit from obfuscating i's beliefs.

Let $w_{j,1}^H$ denote the wholesale price associated with a producer j of capacity K_j^H in the first stage. The following proposition characterizes when it is beneficial for the retailer to deter obfuscation by committing to output reduction.

Proposition 9 *For any $\delta \in [0, 1]$, there exists an output level $q^R(\delta) \in [0, K_j^L]$ and a belief threshold $\bar{\theta}(\delta) \in [0, 1)$, such that for all $\theta \leq \bar{\theta}(\delta)$ the retailer commits ex ante to purchasing $q^R(\delta)$ units in the first stage when producer j's submitted wholesale price is greater than $w_{j,1}^H$ (the price associated with capacity K_j^H), whereby a high-capacity producer j will not underproduce to obfuscate his capacity.*

We note that there is a positive likelihood that producer j is indeed bound by a capacity constraint K_L. In such cases, the retailer incurs a cost in the first stage for pursuing this approach — the cost of ordering a suboptimal quantity (in the context of the first-stage subgame) from producer j. When the prior probability that producer j has low capacity is sufficiently low, the gains outweigh the cost. In effect, the retailer commits ex ante to shrink his dealings with a low-capacity producer j; by doing so, the retailer eliminates j's incentive to obfuscate. Figure 5 illustrates the retailer's profits in each stage when pursuing the strategy of output deterrence relative to the cases of obfuscation and complete information. Subfigure 5(a) is characterized by a relatively high prior probability that producer j is a low type; consequently, the retailer's loss in the first stage from pursuing a strategy of output deterrence is not offset by gains in the second stage from disincentivizing obfuscation. In Subfigure 5(b), the prior probability that j is a low type is relatively small, and the strategy of output deterrence pays off to the retailer in both stages.

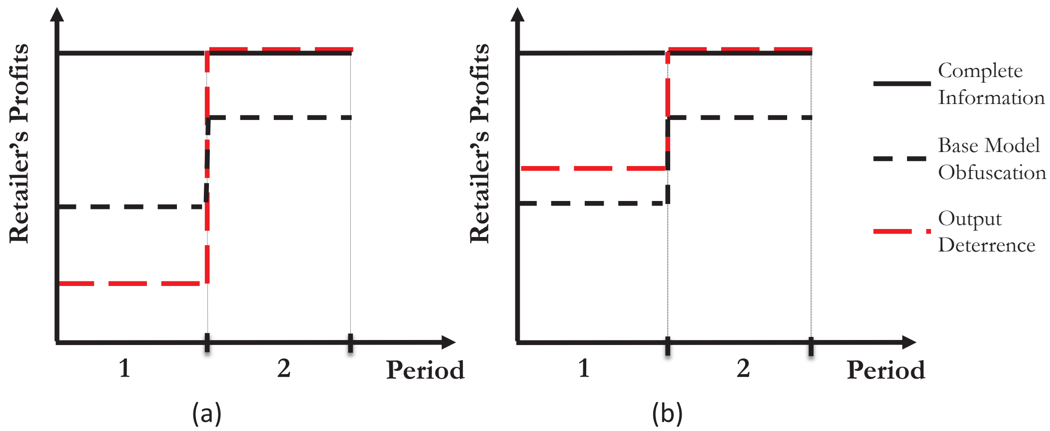

Figure 5: Retailer Profits under Complete Information, Base Model Obfuscation, and Output Deterrence.

However, when the retailer is unable to commit ex ante to output reduction, it is no longer a credible threat for the retailer to reduce the quantity purchased from an apparent low-capacity producer j, and the base model outcome is obtained. This is because, given the structure of the game, producers' wholesale prices are submitted as take-it-or-leave-it offers to the retailer. Once wholesale prices are submitted, they are non-negotiable. Reducing his output order from an apparent low-capacity producer j would decrease the retailer's profit without generating any different action from the producers.[19] Indeed, a promising direction for future work is to consider the possibility of wholesale price (re)negotiations, where producers do not possess all of the bargaining power in determining wholesale prices. It is plausible that an outcome that is more along the lines of the commitment outcome may be obtained as part of a negotiation strategy by the retailer.

Overall, whether pursuing an approach of disclosure or an approach of output reduction, the retailer incurs costs when deterring obfuscation. Interestingly, consumers would benefit from the retailer taking such actions, and so would overall channel profits.

6 Conclusions

In this paper, we study a two-stage supply-chain model with two competing producers, where one of the producers holds private information about his capacity level, which is unknown to his competitor. We show that in some cases, the producer is able to use his competitor's uncertainty about the former's capacity to appear to have a lower capacity than he actually does — manipulating perceptions by strategically cutting production in an earlier stage. As a result, his competitor competes less aggressively, and equilibrium wholesale prices increase.

[19]If the retailer, on the other hand, attempted to order more than K_j^L from producer j, the latter could claim to be capacity constrained at K_j^L.

Our findings indicate that obfuscation has important implications for supply chains. In particular, obfuscation can moderate producers' competition, which enhances producers' profits. The intuition for this result is as follows. If the non-obfuscating producer, i, perceives producer j to be further constrained than j's capacity dictates, he will relax his wholesale pricing strategy. In other words, producer i will price less aggressively, setting a higher wholesale price. Anticipating this behavior, producer j, in turn, will relax his own pricing strategy, increasing his wholesale price as well, bringing the wholesale market closer to a monopolistic outcome. This results in higher wholesale prices, leading the retailer to acquire and sell fewer units at higher retail prices. Consequently, consumers pay more, the retailer's profits shrink, and producers' total profits increase. Overall channel profits decrease.

Most interestingly, we show that as a result of obfuscation, there are situations where both producers' individual profits rise. These situations do not occur when producers actively communicate credible information or attempt to acquire it — on the contrary — they happen when producers do not verify information. More specifically, we show that although producer i fully recognizes the possibility of j obfuscating, there are cases where producer i is better off not acquiring information about j's real capacity constraint. Said another way, the lack of verifiable information about j's true constraint facilitates profit gains for both producers. These findings have direct implications to oligopolistic competition and antitrust laws, since illegal collusion, as defined by the Sherman Act, rests on whether firms have reached an agreement. Hence, the equilibrium outcome in our framework, which can be interpreted as a form of tacit collusion, does not necessarily violate the law. In other words, obfuscation can serve as a proxy for price collusion, especially in situations where such collusion may be deemed illegal or at the very least be perceived in a negative light.

We also show that there are situations where a retailer would choose to deter a producer's capacity obfuscation by either disclosing information regarding capacities to rival producers or by altering its purchasing strategy. Although a retailer's profit is contingent on acquiring goods to sell from the producers, there is a conflict of interest in terms of eliminating collusive behavior. In particular, the retailer may be willing to expend resources in order to prevent obfuscation-induced collusion.

Understanding the economic forces behind a competitor's production and pricing decisions is critical to determining a firm's own production and pricing strategies. Our findings thus have clear managerial implications. First, they elucidate some of the incentives that a producer may have for manipulating capacity perceptions, particularly those that are motivated by competitive pressures. Second, our results indicate that predictions generated from models that ignore the possibility of obfuscation may be distorted, in the sense that these predictions ignore a possibility that is relevant to determining profit-maximizing strategies. Third, our findings indicate that there are cases where a retailer may choose to strategically alter its wholesale purchases in order to reduce capacity-related uncertainty among producers.

There are many interesting questions that remain to be answered. First, it would be instructive to study settings where multiple producers can obfuscate their production capacities. A natural extension to that framework is a setting with both competing retailers, as in [6], and competing producers, where retailers can obfuscate their sales and producers can obfuscate their capacities. It would also be worthwhile to develop a deeper understanding of feasible contracting protocols within the revenue chain, and the approaches that could be used by both retailers and producers to contractually mitigate the effects of obfuscation. One could also develop a more continuous setup in lieu of the discrete prior distribution over types that is used in our current framework. We believe that the topic of obfuscation in a supply chain presents many promising and important opportunities for future research.

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Appendices

A Complete Information Benchmark — Results Under No Obfuscation

We now turn our attention to the producers' problem when there is no private information, i.e., producer j's capacity constraint is common knowledge. The one-shot (Nash) equilibrium wholesale prices must reflect any relevant capacity constraint and the retailer's choice of retail prices as a function of both wholesale prices. It is straightforward to see that an equilibrium of the overall game consists of producers and the retailer pricing according to the one-shot Nash prices in each stage. For a given wholesale price from his competitor and anticipating the retailer's response to both wholesale prices, producer i would solve:

$$\begin{aligned} \max \quad & (w_{i,t} - c_i)\left[\alpha - \left(\frac{\alpha}{2\,(1-\gamma)} + 0.5 w_{i,t}\right) + \gamma\left(\frac{\alpha}{2\,(1-\gamma)} + 0.5 w_{j,t}\right)\right]^{+} \\ \text{s.t.} \quad & w_{i,t} \geq c_i \end{aligned} \tag{7}$$

Here, we substituted for the retailer's optimal prices to highlight the dependence of sales quantities on the wholesale prices, and include producer j's wholesale price, who solves an analogous problem, with the added capacity constraint given by (8):

$$\alpha - \left(\frac{\alpha}{2\,(1-\gamma)} + 0.5 w_{j,t}\right) + \gamma\left(\frac{\alpha}{2\,(1-\gamma)} + 0.5 w_{i,t}\right) \leq K_j \tag{8}$$

where K_j is producer j's capacity, with $K_j \in \{K_j^L, K_j^H\}$.

Producer i's Problem: Taking the derivative of (7) with respect to $w_{i,t}$, and setting it equal to zero, we get producer i's optimal wholesale response function, given by $w_{i,t}(w_{j,t})$:

$$w_{i,t}(w_{j,t}) = \frac{1}{2}c_i + \frac{\alpha + \gamma w_{j,t}}{2} \tag{9}$$

Producer j's Problem: If producer j is capacity constrained, then the Lagrangian problem becomes:

$$\begin{aligned} \max_{w_{j,t},M} L_j \;=\; & (w_{j,t} - c_j)\left[\alpha - \left(\frac{\alpha}{2(1-\gamma)} + 0.5 w_{j,t}\right) + \gamma\left(\frac{\alpha}{2(1-\gamma)} + 0.5 w_{i,t}\right)\right] \\ & - M\left[\alpha - \left(\frac{\alpha}{2(1-\gamma)} + 0.5 w_{j,t}\right) + \gamma\left(\frac{\alpha}{2(1-\gamma)} + 0.5 w_{i,t}\right) - K_j\right] \end{aligned} \tag{10}$$

where M is the Lagrange multiplier. The optimal wholesale price response function for producer j, when he is capacity constrained at K_j, where $K_j \in \{K_j^L, K_j^H\}$, is given by

$$w_{j,t}^{C}(w_{i,t}) = \alpha + \gamma(w_{i,t}) - 2K_j \tag{11}$$

If producer j is not capacity constrained, then the optimal wholesale price response function is:

$$w_{j,t}^{U}(w_{i,t}) = \frac{1}{2}c_j + \frac{\alpha + \gamma(w_{i,t})}{2} \tag{12}$$

Using the wholesale price response function given in Equations (9), (11) and (12), we can solve for the equilibrium wholesale prices, sales quantities, and profits shown in Table 1.

	Equilibrium Capacity Configuration	
	Unconstrained	Constrained
Wholesale Prices	$w_{i,t}^{U^*} = \frac{(2+\gamma)\alpha + 2c_i + \gamma c_j}{4-\gamma^2}$ $w_{j,t}^{U^*} = \frac{(2+\gamma)\alpha + 2c_j + \gamma c_i}{4-\gamma^2}$	$w_{i,t}^{C^*} = \frac{(1+\gamma)\alpha + c_i - 2\gamma K_j}{2-\gamma^2}$ $w_{j,t}^{C^*} = \frac{(2+\gamma)\alpha + \gamma c_i - 4K_j}{2-\gamma^2}$
Demands	$D_{i,t}^{U^*} = \frac{(2+\gamma)\alpha - (2-\gamma^2)c_i + \gamma c_j}{2(4-\gamma^2)}$ $D_{j,t}^{U^*} = \frac{(2+\gamma)\alpha - (2-\gamma^2)c_j + \gamma c_i}{2(4-\gamma^2)}$	$D_{i,t}^{C^*} = \frac{(1+\gamma)\alpha - (1-\gamma^2)c_i - 2\gamma K_j}{2(2-\gamma^2)}$ $D_{j,t}^{C^*} = K_j$
Producer Profits	$\Pi_{i,t}{}^{U^*} = \frac{\left[(2+\gamma)\alpha - (2-\gamma^2)c_i + \gamma c_j\right]^2}{2(4-\gamma^2)^2}$ $\Pi_{j,t}{}^{U^*} = \frac{\left[(2+\gamma)\alpha - (2-\gamma^2)c_j + \gamma c_i\right]^2}{2(4-\gamma^2)^2}$	$\Pi_{i,t}^{C^*} = \frac{\left[(1+\gamma)\alpha - (1-\gamma^2)c_i - 2\gamma K_j\right]^2}{2(2-\gamma^2)^2}$ $\Pi_{j,t}^{C^*} = \frac{\left[(2+\gamma)\alpha + \gamma c_i - (2-\gamma^2)c_j - 4K_j\right]K_j}{2-\gamma^2}$

Table 1: Equilibrium Wholesale Prices, Demands, and Profits Under No Obfuscation

In Table 1, $w_{g,t}^{m^*}, D_{g,t}^{m^*}$, and $\Pi_{g,t}^{m^*}$ represent equilibrium wholesale prices, sales quantities, and profits of producer $g \in \{i, j\}$, in stage $t \in \{1, 2\}$, under *equilibrium* capacity configuration m, where $m \in \{U \equiv \text{ Unconstrained}, \; C \equiv \text{ Constrained}\}$. With the above results, it is straightforward to show that if j's production capacity satisfies $K_j \leq D_{j,t}^{U^*}$, his capacity constraint is binding in equilibrium, whereas j's constraint is not binding for larger K_j values.

B Equilibria in Second Period — Results Under Obfuscation

Because the problem faced by producer j has the same structure as that of the complete information benchmark, analyzed above, we have that the optimal wholesale price response for producer j, when he is capacity constrained at K_j^L, is given by $w_{j,2}^{L,C}(w_{i,2}) = \alpha + \gamma(w_{i,2}) - 2K_j^L$. Similarly, if producer j is not capacity constrained at a capacity level of K_j^L, then the optimal wholesale price response is $w_{j,2}^{L,U}(w_{i,2}) = \frac{1}{2}c_j + \frac{\alpha + \gamma w_{i,2}}{2}$. Analogously, the optimal wholesale price response for producer j, when he is capacity constrained at K_j^H, is given by $w_{j,2}^{H,C}(w_{i,2}) = \alpha + \gamma w_{i,2} - 2K_j^H$. If producer j is not capacity constrained at capacity K_j^H, then the optimal wholesale price response function is $w_{j,2}^{H,U}(w_{i,2}) = \frac{1}{2}c_j + \frac{\alpha + \gamma w_{i,2}}{2}$. As expected, $w_{j,2}^{L,U}(w_{i,2}) = w_{j,2}^{H,U}(w_{i,2})$. That is, when capacity is not binding at either level K_j^H or K_j^L, producer j's response depends only on the wholesale price submitted by producer i.

Using the wholesale price response functions for producers i and j (as given in Appendix A), we can solve for the second-stage equilibrium wholesale prices when producer j's is of type K_j^H and K_j^L, respectively, as shown in Table 2.

Equilibrium Capacity Configuration	UU	$w_{i,2}^{UU^*} = \dfrac{(2+\gamma)\alpha + 2c_i + \gamma c_j}{4-\gamma^2}$ $w_{j,2}^{H,UU^*} = w_{j,2}^{L,UU^*} = \dfrac{(2+\gamma)\alpha + 2c_j + \gamma c_i}{4-\gamma^2}$
	UC	$w_{i,2}^{UC^*} = \dfrac{[2+\gamma(1+\theta_2)]\,\alpha + 2c_i + \gamma(1-\theta_2)\,c_j - 4\theta_2\gamma K_j^L}{4-\gamma^2(1+\theta_2)}$ $w_{j,2}^{H,UC^*} = \dfrac{(2+\gamma)\alpha + \gamma c_i + (2-\theta_2\gamma^2)c_j - 2\theta_2\gamma^2 K_j^L}{4-\gamma^2(1+\theta_2)}$ $w_{j,2}^{L,UC^*} = \dfrac{(4+2\gamma)\alpha + 2\gamma c_i + \left[\gamma^2(1-\theta_2)\right]c_j - 2\left[4-\gamma^2(1-\theta_2)\right]K_j^L}{4-\gamma^2(1+\theta_2)}$
	CC	$w_{i,2}^{CC^*} = \dfrac{(1+\gamma)\alpha + c_i - 2\gamma K_j^H + 2\theta_2\gamma\left(K_j^H - K_j^L\right)}{2-\gamma^2}$ $w_{j,2}^{H,CC^*} = \dfrac{(2+\gamma)\alpha + \gamma c_i - 4K_j^H + 2\theta_2\gamma^2\left(K_j^H - K_j^L\right)}{2-\gamma^2}$ $w_{j,2}^{L,CC^*} = \dfrac{(2+\gamma)\alpha + \gamma c_i - 4K_j^L - 2(1-\theta_2)\gamma^2\left(K_j^H - K_j^L\right)}{2-\gamma^2}$

Table 2: Wholesale Prices in the Second Stage

Second Period Equilibrium Demands

For each of the three configurations given in Table 2, we substitute the relevant equilibrium wholesales prices into (13) and (14) to determine second-stage equilibrium sales quantities:

$$D_{i,2}\left(p_{i,2}^*, p_{j,2}^*\right) = \alpha - p_{i,2}^*(w_{i,2}) + \gamma p_{j,2}^*(w_{j,2}) \tag{13}$$

$$D_{j,2}\left(p_{i,2}^*, p_{j,2}^*\right) = \alpha - p_{j,2}^*(w_{j,2}) + \gamma p_{i,2}^*(w_{i,2}) \tag{14}$$

where $p_{i,2}^*(w_{i,2}) = \dfrac{\alpha}{2(1-\gamma)} + 0.5w_{i,2}$ and $p_{j,2}^*(w_{j,2}) = \dfrac{\alpha}{2(1-\gamma)} + 0.5w_{j,2}$, as detailed in (2).

Second-Stage Demands under (U,U) Configuration: These are the producers' sales quantities when producer j is unconstrained in equilibrium whether he is of type K_j^H or K_j^L.

$$D_{i,2}^{UU^*} = D_{i,2}^{H,UU^*} = D_{i,2}^{L,UU^*} = \frac{(2+\gamma)\alpha - (2-\gamma^2)c_i + \gamma c_j}{2(4-\gamma^2)} \tag{15}$$

$$D_{j,2}^{UU^*} = D_{j,2}^{H,UU^*} = D_{j,2}^{L,UU^*} = \frac{(2+\gamma)\alpha - (2-\gamma^2)c_j + \gamma c_i}{2(4-\gamma^2)} \tag{16}$$

In other words, in equilibrium, capacities are such that $K_j^H \geq D_{j,2}^{UU^*}$ and $K_j^L \geq D_{j,2}^{UU^*}$.

Second-Stage Demands under (U,C) Configuration: These sales quantities represent the case where a producer j of type K_j^H is not constrained in equilibrium, but where a producer j of type K_j^L would be. The sales quantities given by (17)-(18) and (19)-(20) represent the producers' equilibrium sales when producer j submits wholesale prices $w_{j,2}^{H,UC^*}$ and $w_{j,2}^{L,UC^*}$, respectively.

$$D_{i,2}^{H,UC^*} = \frac{[2+\gamma(1-\theta_2-\theta_2\gamma)]\,\alpha-(2-\gamma^2)c_i+\gamma\left(1+\theta_2-\theta_2\gamma^2\right)c_j+2\theta_2\gamma(2-\gamma^2)K_j^L}{2\left[4-\gamma^2(1+\theta_2)\right]} \quad (17)$$

$$D_{j,2}^{H,UC^*} = \frac{(2+\gamma)\alpha-(2-\gamma^2)c_j+\gamma c_i-2\theta_2\gamma^2K_j^L}{2\left[4-\gamma^2(1+\theta_2)\right]} \quad (18)$$

and

$$D_{i,2}^{L,UC^*} = \frac{[2+\gamma(3-\theta_2+\gamma(1-\theta_2))]\,\alpha-2(1-\gamma^2)c_i-\gamma\left[(1-\gamma^2)(1-\theta_2)\right]c_j-2\gamma\left[4-2\theta_2-\gamma^2(1-\theta_2)\right]K_j^L}{2(4-\gamma^2(1+\theta_2))} \quad (19)$$

$$D_{j,2}^{L,UC^*} = K_j^L \quad (20)$$

Second-Stage Demands under (C, C) Configuration: These sales quantities represent the case where a producer j of either type is constrained in equilibrium.

$$D_{i,2}^{H,CC^*} = \frac{(1+\gamma)\alpha-(1-\gamma^2)c_i-2\theta_2\gamma(1-\gamma^2)(K_j^H-K_j^L)-2\gamma K_j^H}{2(2-\gamma^2)} \quad (21)$$

$$D_{j,2}^{H,CC^*} = K_j^H \quad (22)$$

and

$$D_{i,2}^{L,CC^*} = \frac{(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^L+2\gamma\left[(1-\gamma^2)(1-\theta_2)\right](K_j^H-K_j^L)}{2(2-\gamma^2)} \quad (23)$$

$$D_{j,2}^{L,CC^*} = K_j^L \quad (24)$$

Several observations are in order. First, we note that $D_{j,2}^{H,UC^*}$ is decreasing in K_j^L. The intuition for this is simple. As K_j^L increases, producer i prices more aggressively (i.e., i sets a lower wholesale price $w_{i,2}^{UC^*}$). Consequently, the sales quantity for j's product drops. Second, for $K_j^L \to 0$, for any $\theta_2 > 0$, $D_{j,2}^{H,UC^*} > D_{j,2}^{UU^*}$. The intuition here is similar (but in reverse) — appearing to have a low capacity works to the benefit of producer j because i prices less aggressively (i.e., i sets a higher wholesale price $w_{i,2}^{H,UC^*} > w_{i,2}^{UU^*}$) when he believes that j's production capacity is severely constrained. Third, at $K_j^L = D_{j,2}^{UU^*}$, we have $D_{j,2}^{H,UC^*} = D_{j,2}^{UU^*}$, i.e., equilibrium sales quantities are the same as in the unconstrained case.

B.1 Proof of Lemma 1:

We first consider the case where producers of either type are unconstrained in equilibrium. That is, suppose $K_j^L \geq D_{j,t}^{UU^*} = D_{j,t}^{L,UU^*}$ and $K_j^H \geq D_{j,t}^{UU^*} = D_{j,t}^{H,UU^*}$. Because the equilibrium wholesale prices $(w_{i,t}^{UU^*}, w_{j,t}^{H,UU^*} = w_{j,t}^{L,UU^*})$, as well as equilibrium sales given by (15) and (16) remain unchanged, whether a K_j^L type producer pretends to be a K_j^H type or a K_j^H type pretends to have higher capacity, producers have no incentive to obfuscate with a level greater than their true capacity under the (U, U) configuration. Under capacity configuration (U, C), for any fixed $K_j^H \geq D_{j,t}^{H,UU^*}$, a K_j^L-type producer pretending to be of type K_j^H shifts the producer to the (U, U) equilibrium discussed above. Under capacity configuration (C, C), by pretending to have higher capacity (e.g., having a K_j^L-type producer pretend to be a K_j^H type) would lead the producer, already constrained in equilibrium, to sell the same quantity at a lower wholesale price. This is because $w_{j,t}^{H,CC^*}$ decreases in the difference of $\left(K_j^H - K_j^L\right)$ and $w_{j,t}^{L,CC^*}$ decreases in K_j^L more so than it increases in $\left(K_j^H - K_j^L\right)$. Therefore, a producer j does not benefit from appearing to have a capacity level that is higher than his true capacity.

Derivation of Second-Stage Equilibrium Profits

We next derive the producers' *ex-post* profits under each of the equilibrium capacity configurations in the second stage. These are *ex-post* profits in the sense that they are obtained at the end of the game where producer i's uncertainty regarding j's capacity is resolved.

Profits under (U,U) configuration: Because producer j submits the same wholesale price under either capacity level, we can readily find second-stage equilibrium profits for both producers under the (U,U) configuration. Using the equilibrium wholesale prices given in Table 2, profits are given by

$$\Pi_{i,2}^{UU^*} = \frac{\left[(2+\gamma)\alpha - (2-\gamma^2)c_i + \gamma c_j\right]^2}{2(4-\gamma^2)^2} \tag{25}$$

$$\Pi_{j,2}^{UU^*} = \Pi_{j,2}^{H,UU^*} = \Pi_{j,2}^{L,UU^*} = \frac{\left[(2+\gamma)\alpha - (2-\gamma^2)c_j + \gamma c_i\right]^2}{2(4-\gamma^2)^2} \tag{26}$$

Profits under (U,C) configuration: Let $\Pi_{g,2}^{H,UC^*}$ and $\Pi_{g,2}^{L,UC^*}$ represent producer g's, $g \in \{i,j\}$, expected profits under the (U,C) configuration. If producer j chooses to submit $w_{j,2}^{H,UC^*}$ as his wholesale price, then profits can obtained from $\Pi_{j,2}^{H,UC^*} = \left(w_{j,2}^{H,UC^*} - c_j\right) \cdot D_{j,2}^{H,UC^*}$ and $\Pi_{i,2}^{H,UC^*} = \left(w_{i,2}^{UC^*} - c_i\right) \cdot D_{i,2}^{H,UC^*}$. Substituting the relevant expressions, we have

$$\Pi_{i,2}^{H,UC^*} = \frac{\begin{array}{c}\left[(2+\gamma(1-\theta_2-\theta_2\gamma))\,\alpha - (2-\gamma^2)c_i + \gamma(1+\theta_2-\theta_2\gamma^2)c_j + 2\theta_2\gamma(2-\gamma^2)K_j^L\right] \cdot \\ \left[\left[2+\gamma(1+\theta_2)\right]\alpha + \gamma(1-\theta_2)c_j - \left(2-\gamma^2(1+\theta_2)\right)c_i - 4\theta_2\gamma K_j^L\right]\end{array}}{2\left(4-\gamma^2(1+\theta_2)\right)^2} \tag{27}$$

$$\Pi_{j,2}^{H,UC^*} = \frac{\left[(2+\gamma)\alpha - (2-\gamma^2)c_j + \gamma c_i - 2\theta_2\gamma^2 K_j^L\right]^2}{2\left(4-\gamma^2(1+\theta_2)\right)^2} \tag{28}$$

On the other hand, if producer j chooses to submit $w_{j,2}^{L,UC^*}$ as his wholesale price, then profits are determined from $\Pi_{i,2}^{L,UC^*} = \left(w_{i,2}^{UC^*} - c_i\right) \cdot D_{i,2}^{L,UC^*}$ and $\Pi_{j,2}^{L,UC^*} = \left(w_{j,2}^{L,UC^*} - c_j\right) \cdot D_{j,2}^{L,UC^*}$. Substituting the relevant expressions gives

$$\Pi_{i,2}^{L,UC^*} = \frac{\begin{array}{c}\left[\left[2+\gamma\left(3-\theta_2+\gamma(1-\theta_2)\right)\right]\alpha - 2(1-\gamma^2)c_i - \gamma\left((1-\theta_2)-\gamma^2(1-\theta_2)\right)c_j \right.\\ \left. -2\gamma\left(4-2\theta_2-\gamma^2(1-\theta_2)\right)K_j^L\right] \cdot \\ \left[\left[2+\gamma(1+\theta_2)\right]\alpha + \gamma(1-\theta_2)c_j - \left(2-\gamma^2(1+\theta_2)\right)c_i - 4\theta_2\gamma K_j^L\right]\end{array}}{2\left(4-\gamma^2(1+\theta_2)\right)^2} \tag{29}$$

$$\Pi_{j,2}^{L,UC^*} = \frac{2\left[(2+\gamma)\alpha - (2-\gamma^2)c_j + \gamma c_i - \left(4-\gamma^2(1-\theta_2)\right)K_j^L\right]K_j^L}{4-\gamma^2(1+\theta_2)} \tag{30}$$

Profits under (C,C) configuration: Let $\Pi_{g,2}^{H,CC^*}$ and $\Pi_{g,2}^{L,CC^*}$ denote producer g's, $g \in \{i,j\}$, expected profits under the (C,C) configuration. As in the previous configuration, if producer j

chooses to submit $w_{j,2}^{H,CC^*}$, then *ex-post* profits are:

$$\Pi_{i,2}^{H,CC^*} = \frac{\begin{array}{l}\left[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^H-2\theta_2\gamma(1-\gamma^2)\left(K_j^H-K_j^L\right)\right]\cdot\\ \left[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^H+2\theta_2\gamma(K_j^H-K_j^L)\right]\end{array}}{2\left[2-\gamma^2\right]^2} \tag{31}$$

$$\Pi_{j,2}^{H,CC^*} = \frac{\left[(2+\gamma)\alpha+\gamma c_i-(2-\gamma^2)c_j-4K_j^H+2\theta_2\gamma^2\left(K_j^H-K_j^L\right)\right]K_j^H}{2-\gamma^2} \tag{32}$$

Alternatively, if producer j submits $w_{j,2}^{L,CC^*}$ as his wholesale price, then profits are:

$$\Pi_{i,2}^{L,CC^*} = \frac{\begin{array}{l}\left[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^H+2\theta_2\gamma(K_j^H-K_j^L)\right]\cdot\\ \qquad\left[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^L+2(\gamma(1-\gamma^2)(1-\theta_2))\left(K_j^H-K_j^L\right)\right]\end{array}}{2(2-\gamma^2)^2} \tag{33}$$

$$\Pi_{j,2}^{L,CC^*} = \frac{\left[(2+\gamma)\alpha+\gamma c_i-(2-\gamma^2)c_j-4K_j^L-2\gamma^2(1-\theta_2)(K_j^H-K_j^L)\right]K_j^L}{2-\gamma^2} \tag{34}$$

B.2 Proof of Proposition 1:

We begin by analyzing the profits of a K_j^H-type producer j for each capacity configuration. To prove Proposition 1, we see that under the (U,C) configuration, the partial derivative with respect to θ, of producer j's profit function, given by (28), is:

$$\frac{\partial \Pi_{j,2}^{H,UC^*}}{\partial \theta} = \underbrace{\frac{1}{\left[\gamma^2(1+\theta)-4\right]^3}}_{\leq 0}\cdot\left[\begin{array}{l}\overbrace{-\left[(2+\gamma)\alpha+\gamma c_i-(2-\gamma^2)c_j-2\theta\gamma^2K_j^L\right]}^{\leq 0}\\ \cdot\gamma^2\underbrace{\left[(2+\gamma)\alpha+\gamma c_i-(2-\gamma^2)c_j-2(4-\gamma^2)K_j^L\right]}_{\geq 0}\end{array}\right]\geq 0$$

We must have from $w_{j,2}^{H,UC}\geq c_j$, where $w_{j,2}^{H,UC}$ is given in Table 2; this implies $(2+\gamma)\alpha+\gamma c_i-(2-\gamma^2)c_N-2\theta\gamma^2K_j^L\geq 0$. Hence, the first term in the bracketed expression is non-positive. Under the UC configuration $K_j^L\leq\dfrac{(2+\gamma)\alpha+\gamma c_i-(2-\gamma^2)c_j}{2(4-\gamma^2)}=D_{j,2}^{UU^*}$ Thus, the second term in brackets is non-negative. Therefore, we conclude that a high-capacity producer j's profits are increasing in θ under the (U,C) configuration.

For the CC configuration, the partial derivative of (32) is $\dfrac{\partial\Pi_{j,2}^{H,CC^*}}{\partial\theta}=\dfrac{2\gamma^2\left(K_j^H-K_j^L\right)K_j^H}{2-\gamma}\geq 0.$ This result indicates that producer j's profits are increasing in θ, under the (C,C) configuration.

B.3 Proof of Proposition 2:

We next consider the effect of θ on producer i's profits. Under the UC configuration, setting $\frac{\partial\Pi_{i,2}^{H,UC^*}}{\partial\theta}=0$, we find the stationary point:

$$\widetilde{\theta} = \frac{\gamma\left[(2+\gamma)\alpha+\gamma c_j-(2-\gamma^2)c_i\right]}{[4+\gamma(2-\theta)]\alpha+(2-\gamma^2)\gamma c_i-(4-3\gamma^2)c_j-8(2-\gamma^2)K_j^L}$$

Thus, for $K_j^L \leq \frac{\alpha - c_j + \gamma c_i}{4}$, producer i's profit is increasing up to $\theta^* = \widetilde{\theta}$, and decreasing thereafter. For $K_j^L > \frac{\alpha - c_j + \gamma c_i}{4}$, producer i's profit are strictly increasing in θ.

Under the (C, C) configuration, the first and second derivatives of (31) with respect to θ are:

$$\frac{\partial \Pi_{i,2}^{H,CC^*}}{\partial \theta} = \frac{\gamma^2 \left(K_j^H - K_j^L\right) \left[(1+\gamma)\gamma\alpha - (1-\gamma^2)\gamma c_i - 2\gamma^2 K_j^H - 4\theta(1-\gamma^2)(K_j^H - K_j^L)\right]}{(2-\gamma^2)^2}$$

$$\frac{\partial^2 \Pi_{i,2}^{H,CC^*}}{\partial \theta^2} = \frac{4\gamma^2 \left(K_j^H - K_j^L\right)^2 \left[\gamma^2 - 1\right]}{(2-\gamma^2)^2} \leq 0$$

Setting the first derivative to zero, we get $\theta^{CC^*} = \frac{\gamma \left[(1+\gamma)\alpha - (1-\gamma^2)c_i - 2\gamma K_j^H\right]}{4\left[(1-\gamma^2)(K_j^H - K_j^L)\right]}$ as the θ value that maximizes producer i's profits, if it falls in the interval $[0, 1]$. If K_j^L satisfies the following condition,

$$K_j^L \geq \frac{-\left[(1+\gamma)\gamma\alpha - (1-\gamma^2)\gamma c_i - (4-2\gamma^2)K_j^H\right]}{4\left[1-\gamma^2\right]} \tag{35}$$

then producer i's profits are guaranteed to increase in θ, for $K_j^H \leq D_{j,2}^{UC^*}$ under the (C, C) configuration, as stated in Proposition 2.

B.4 Proof of Proposition 3:

Under the UC configuration, taking the derivative of the sum of the producers' profit functions with respect to θ, we get

$$\frac{\partial\left(\Pi_{i,2}^{H,UC^*} + \Pi_{j,2}^{H,UC^*}\right)}{\partial \theta} =$$

$$\underbrace{\frac{1}{2\left[-4+\gamma^2(1+\theta)\right]^3}}_{\leq 0} \left[\begin{array}{c} -\gamma^2 \cdot \overbrace{\left[(2+\gamma)\alpha + \gamma c_i - (2-\gamma^2)c_j - 2\left(4-\gamma^2\right)K_j^L\right]}^{\geq 0} \\ \cdot \underbrace{\left[\begin{array}{c} \left[(2+\gamma)^2 - \left[4+(2-\gamma)\gamma\right]\theta\right]\alpha + 4\theta\left[4-3\gamma^2\right]K_j^L \\ -(1-\theta)(4-3\gamma^2)c_j + \left[\gamma^3 - \gamma(2-\gamma^2)\theta\right]c_i \end{array}\right]}_{\geq 0} \end{array}\right] \tag{36}$$

To show that the last term in the bracketed expression is non-negative, we will demonstrate that the positive components, at their least positive values, are greater than the negative components at their most negative values. Separating the components of the third term based on whether they are positive or negative, we get

Negative	Positive
$-[4+(2-\gamma)\gamma]\theta\alpha$	$(2+\gamma)^2\alpha$
$-(1-\theta)[4-3\gamma^2]c_j$	$4\theta\left[4-3\gamma^2\right]K_j^L$
$-\gamma(2-\gamma^2)\theta c_i$	$\gamma^3 c_i$

The sum of the positive components is least positive at $(2+\gamma)^2\alpha + \gamma^3 c_i$, when $\theta = 0$ and $K_j^L = 0$. Now consider the negative parts of the third term, and let $A \equiv [4+(2-\gamma)\gamma]\alpha + \gamma(2-\gamma^2)c_i$ and

$B \equiv (4-3\gamma^2)c_j$, where the sum of negative terms equals $-(\theta A + (1-\theta)B)$. We first note that $A \geq B$, for all γ values. This follows from the fact that $c_j \leq \alpha$, which implies an upper bound for $B \leq (4-3\gamma^2)\alpha$. Similarly, when $c_i = 0$, we have a lower bound for $A \geq [4+(2-\gamma)\gamma]\alpha = (4+2\gamma-\gamma^2)\alpha$. Thus, $A \geq B$.

Given that $A \geq B$, the negative components will be their most negative (from $-(\theta A + (1-\theta)B)$) when $\theta = 1$, equaling $-([4+(2-\gamma)\gamma]\alpha + \gamma(2-\gamma^2)c_i)$. Finally, we note that the magnitude of the negative components at their most negative ($[4+(2-\gamma)\gamma]\alpha+\gamma(2-\gamma^2)c_i$ is always smaller than that of the positive components at the least positive value, as $(2+\gamma)^2\alpha+\gamma^3 c_i = (4+4\gamma+\gamma^2)\alpha+\gamma^3 c_i$. Therefore, it follows that the third term in brackets is non-negative, leading to the desired result. Under the CC configuration, taking the derivative of the sum of the producers profit function with respect to θ, we get

$$\frac{\partial\left(\Pi_{i,2}^{H,CC^*} + \Pi_{j,2}^{H,CC^*}\right)}{\partial\theta} = \underbrace{\frac{\gamma^2}{(2-\gamma^2)^2}}_{\geq 0} \underbrace{\left[\begin{array}{l} (1+\gamma)\gamma\left[\alpha-(1-\gamma)c_i\right](K_j^H - K_j^L) \\ +4(1-\gamma^2)(K_j^H - K_j^L)\left[K_j^H - \theta(K_j^H - K_j^L)\right] \end{array}\right]}_{\geq 0} \geq 0$$

Therefore, under the CC configuration, the sum of producers' second-stage profits is increasing in θ.

B.5 Proof of Proposition 4:

It is straightforward to see that the wholesale prices under the CC configuration (Table 2) are linearly increasing in θ. The partial derivatives of the wholesale prices with respect to θ, under the UC configuration, are as follows:

$$\frac{\partial w_{i,2}^{UC^*}}{\partial\theta} = \frac{2\gamma\left[(2+\gamma)\alpha+\gamma c_i-(2-\gamma^2)c_j-2\left(4-\gamma^2\right)K_j^L\right]}{\left[4-\gamma^2(1+\theta)\right]^2} \geq 0$$

$$\frac{\partial w_{j,2}^{H,UC^*}}{\partial\theta} = \frac{\gamma^2\left[(2+\gamma)\alpha+\gamma c_i-(2-\gamma^2)c_j-2\left(4-\gamma^2\right)K_j^L\right]}{\left[4-\gamma^2(1+\theta)\right]^2} \geq 0$$

Hence, we conclude that the producers wholesale prices are non-decreasing in θ. Additionally, because the retail prices are an increasing transformation of the wholesale prices, i.e., $\frac{\partial p_{g,2}^{UC^*}}{\partial w_{g,2}^{H,UC^*}} = \frac{\partial p_{g,2}^{CC^*}}{\partial w_{g,2}^{H,CC^*}} = \frac{1}{2}, g \in \{i,j\}$, this preserves the non-decreasing property in θ. In other words, $\frac{\partial p_{g,2}}{\partial\theta} = \frac{\partial p_{g,2}}{\partial w_{g,2}} \cdot \frac{\partial w_{g,2}}{\partial w_{g,2}}$. Therefore, we conclude the retail prices are also non-decreasing in θ.

Retailer's Equilibrium Profits:

The retailer's profit functions under the (U,U), (U,C) and (C,C) configurations, respectively, are:

$$\Pi_R^{UU^*} = \frac{2[4+\gamma(4+\gamma)]\alpha^2 - 2[4-\gamma^2(3+\gamma)]\alpha(c_i+c_j) - 2\gamma^3(1-\gamma)c_ic_j + (1-\gamma)(4-3\gamma^2)(c_i^2+c_j^2)}{4(4-\gamma^2)^2(1-\gamma)} \quad (37)$$

$$\Pi_R^{UC^*} = \frac{\left[\begin{array}{l} (3\gamma^3 - 3\gamma^2 - 4\gamma + 4)c_i^2 + [(16\theta\gamma^2 + 12\theta\gamma^3 - 16\theta\gamma - 12\theta\gamma^4)K_j^L \\ \quad +(-4\theta\gamma + 2\gamma^4 + 4\theta\gamma^2 - 2\gamma^3 + 4\theta\gamma^3 - 4\theta\gamma^4)c_j + (-4\theta\gamma^3 + 4\theta\gamma + 6\gamma^2 - 8 + 2\gamma^3)\alpha]c_i \\ \quad +(-12\theta^2\gamma^4 + 12\theta^2\gamma^5 + 16\theta^2\gamma^2 - 16\theta^2\gamma^3)\left(K_j^L\right)^2 \\ \quad +[(8\theta^2\gamma^2 - 8\theta^2\gamma^4 - 4\theta\gamma^5 + 8\theta^2\gamma^5 + 4\theta\gamma^4 - 8\theta^2\gamma^3)c_j \\ \quad +(16\theta\gamma + 8\theta^2\gamma^4 - 12\theta\gamma^3 - 4\theta\gamma^4 - 8\theta^2\gamma^2)\alpha]K_j^L \\ \quad +(3\gamma^3 + 2\theta\gamma^3 + 2\theta\gamma^4 + \theta^2\gamma^2 + \theta^2\gamma^5 - 2\theta\gamma^2 - \theta^2\gamma^4 - 3\gamma^2 + 4 - \theta^2\gamma^3 - 4\gamma - 2\theta\gamma^5)c_j^2 \\ \quad +(2\theta^2\gamma^4 - 4\theta\gamma^4 + 4\theta\gamma + 2\gamma^3 - 4\theta\gamma^3 - 8 + 4\theta\gamma^2 - 2\theta^2\gamma^2 + 6\gamma^2)\alpha c_j \\ \quad +(8 - 6\theta\gamma^2 - 4\theta\gamma + 8\gamma - 2\theta\gamma^3 + \theta^2\gamma^3 + \theta^2\gamma^2 + 2\gamma^2)\alpha^2 \end{array}\right]}{4(1-\gamma)(4-\gamma^2(1+\theta))^2} \tag{38}$$

$$\Pi_R^{CC^*} = \frac{\left[\begin{array}{l} \left[\begin{array}{l} -8\theta\gamma^2 + 8\theta\gamma^3 - 8\theta\gamma^5 - 4\theta^2\gamma^3 - 4\theta^2\gamma^4 + 4\theta^2\gamma^2 + 4\theta^2\gamma^5 + 8\theta\gamma^4 \\ \quad +12\gamma^3 - 16\gamma - 12\gamma^2 + 16 \end{array}\right]\left(K_j^H\right)^2 \\ + \left[\begin{array}{l} (4\theta\gamma^3 - 4\gamma^3 - 4\theta\gamma + 4\gamma)\alpha \\ \quad +(8\theta\gamma^2 + 8\theta^2\gamma^4 - 8\theta^2\gamma^2 + 8\theta\gamma^5 + 8\theta^2\gamma^3 - 8\theta\gamma^3 - 8\theta\gamma^4 - 8\theta^2\gamma^5)K_j^L \\ \quad +(4\gamma^2 + 4\gamma^3 + 4\theta\gamma^4 - 4\gamma - 4\theta\gamma^2 - 4\theta\gamma^3 - 4\gamma^4 + 4\theta\gamma)c_i \end{array}\right]K_j^H \\ +(-4\theta^2\gamma^4 + 4\theta^2\gamma^2 - 4\theta^2\gamma^3 + 4\theta^2\gamma^5)\left(K_j^L\right)2 \\ +[(-4\theta\gamma + 4\theta\gamma^2 - 4\theta\gamma^4 + 4\theta\gamma^3)c_i + (-4\theta\gamma^3 + 4\theta\gamma)\alpha]K_j^L \\ +(1+\gamma)\alpha^2 + (-\gamma^2 - \gamma + 1 + \gamma^3)c_i^2 + (-2 + 2\gamma^2)\alpha c_i \end{array}\right]}{4(1-\gamma)(2-\gamma^2)^2} \tag{39}$$

We next consider the effect of θ on channel profits under the (C, C) configuration. The equilibrium channel profits under the (C, C) configuration are $\Pi_{Channel}^{CC^*}$

$$= \frac{\left[\begin{array}{l} -\left[3 + 3\gamma - 2\gamma^2(1+\gamma)\right]\alpha^2 \\ +2\alpha\left[\begin{array}{l} \left[3 - 5\gamma^2 + 2\gamma^4\right]c_i \\ -2\left[K_j^L\gamma(-1+\gamma^2)^2\theta + K_j^H(4 - 4\gamma^2 + \gamma^4 - \gamma^5\theta + 2\gamma^3(1+\theta) - \gamma(3+\theta))\right] \end{array}\right] \\ -(1-\gamma)\left[\begin{array}{l} c_i^2\left[3 - 5\gamma^2 + 2\gamma^4\right] + 4c_i\gamma\left[(-K_j^L)(-1+\gamma^2)^2\theta + K_j^H(3 + \theta + \gamma^4\theta - 2\gamma^2(1+\theta))\right] \\ -4\left[\begin{array}{l} -c_j(2-\gamma^2)^2K_j^H + 2K_j^L\gamma^2(1-\gamma^2)K_j^H(1-\theta)\theta + \left(K_j^L\right)^2\gamma^2(1-\gamma^2)\theta^2 \\ + \left(K_j^H\right)^2(4 - \gamma^4(-2+\theta)\theta + \gamma^2(-3 - 2\theta + \theta^2)) \end{array}\right] \end{array}\right] \end{array}\right]}{-4(1-\gamma)\left[2-\gamma^2\right]^2} \tag{40}$$

Taking the derivative of (40), with respect to θ, we get

$$\frac{\partial\Pi_{Channel}^{CC^*}}{\partial\theta} = -\frac{(1-\gamma^2)(K_j^H - K_j^L)\gamma\left[(1+\gamma)\alpha - (1-\gamma^2)c_i - 2\gamma K_j^H + 2\theta\gamma(K_j^H - K_j^L)\right]}{(2-\gamma^2)^2} \leq 0$$

Thus, channel profits are always decreasing in θ under the (C, C) configuration.

For the (U,C) configuration, the equilibrium channel profits are given by $\Pi^{UC*}_{Channel}$

$$= \frac{\left[\begin{array}{l} \alpha^2\left[-24-4\gamma(2-\theta)-2\gamma^4\theta(1+\theta)+\gamma^3(4-\theta^2)+\gamma^2(10+10\theta+\theta^2)\right]\alpha^2 \\ +2\alpha(1-\gamma)\left[\begin{array}{l} 2K_j^L\gamma\theta(-4+3\gamma^2-2\gamma\theta+\gamma^3(1+\theta)) \\ +c_i\left[12+2\gamma(2-\theta)+\gamma^4\theta(1+\theta)-\gamma^2(5+6\theta)-\gamma^3(2+\theta-\theta^2)\right] \\ +c_j(12-2\gamma(-2+\theta)+\gamma^3(-2+\theta)+\gamma^4\theta(1+\theta)-\gamma^2(5+4\theta+\theta^2)) \end{array}\right] \\ -(1-\gamma)\left[4\left(K_j^L\right)^2\gamma^2(-4+3\gamma^2)\theta^2-4c_iK_j^L\gamma\theta\left[4+\gamma^4(1+\theta)-\gamma^2(3+2\theta)\right]\right. \\ \quad +c_i^2\left[12+2\gamma^4(1+\theta)-\gamma^2(9+4\theta)\right]+c_j^2\left[12+\gamma^4(2+\theta^2)-\gamma^2(9+2\theta+\theta^2)\right] \\ \quad \left.-2c_j\gamma\left[-2K_j^L\gamma(-2+\gamma^2)(-2+\theta)\theta+c_i(\gamma^4\theta(1+\theta)+2(4+\theta)-\gamma^2(3+5\theta+\theta^2))\right]\right] \end{array}\right]}{-4(1-\gamma)\left[4-\gamma^2(1+\theta)\right]^2} \tag{41}$$

The derivative of (41), with respect to θ, is

$$\frac{\partial\Pi^{UC*}_{Channel}}{\partial\theta} = \frac{\left[\begin{array}{l} \overbrace{\gamma\left[(2+\gamma)\alpha+\gamma c_i-(2-\gamma^2)c_j-2\left[4-\gamma^2\right]K_j^L\right]}^{\geq 0} \\ \quad\cdot\left[\begin{array}{l} \left[4+2\theta\gamma-3\gamma^2-\gamma^3(1+\theta)\right]\alpha+\gamma(2-\theta)(2-\gamma^2)c_j \\ \quad-\left[4-\gamma^2(3+2\theta)+\gamma^4(1+\theta)\right]c_i-2\theta\gamma\left[4-3\gamma^2\right]K_j^L \end{array}\right] \end{array}\right]}{\underbrace{2\left[\gamma^2(1+\theta)-4\right]^3}_{\leq 0}} \tag{42}$$

Channel profits are decreasing in θ if the third term in the bracketed expression is positive. A sufficient condition to ensure profits are decreasing in θ is obtained by setting $K_j^L = D_j^{UU*}$, and is given by

$$\frac{-\left[(1-\gamma)(2+\gamma)^2\alpha+2\gamma(2-\gamma^2)c_j-\left[4-\gamma^2(3-\gamma^2)\right]c_i\right]\left[4-\gamma^2(1+\theta)\right]}{4-\gamma^2} \geq 0$$

Since c_j can be arbitrary, a simpler (though more restrictive) sufficient condition is given by $\alpha(1-\gamma)(2+\gamma)^2-(4-\gamma^2(3-\gamma^2))c_i \geq 0$, which simplifies to $\alpha \geq \dfrac{4-3\gamma^2+\gamma^4}{4-3\gamma^2-\gamma^3}c_i$. From

$$\frac{4-3\gamma^2+\gamma^4}{4-3\gamma^2-\gamma^3} \leq \frac{4-3\gamma^2+\gamma^4}{4-4\gamma^2} \leq \frac{4-3\gamma^4+\gamma^4}{4-4\gamma^2} \leq \frac{1}{1-\gamma^2}$$

it follows that a weaker, though simpler sufficient condition is given by $\alpha \geq \frac{1}{1-\gamma^2}c_i$.

B.6 Proof of Proposition 5:

Under the assumption that producer i anticipates production at K_j^L, if δ is sufficiently large such that

$$\begin{aligned} \delta &\geq \frac{\Pi^H_{j,1}(K_j^H,1)-\Pi^H_{j,1}(K_j^L,1)}{\Pi^H_{j,2}(K_j^H,\theta)-\Pi^H_{j,2}(K_j^H,0)} = \frac{\Pi^{H,CC*}_{j,1}(\theta=1)-\Pi^{L,CC*}_{j,1}(\theta=1)}{\Pi^{H,CC*}_{j,2}(\theta)-\Pi^{H,CC*}_{j,2}(\theta=0)} \\ &= \frac{(2+\gamma)\alpha+\gamma c_i-(2-\gamma^2)c_j+2\gamma^2K_j^H-4(K_j^H+K_j^L)}{2\theta\gamma^2K_j^H} \equiv \underline{\delta} \end{aligned}$$

When $\underline{\delta} \in [0,1]$, a producer j with capacity K_j^H obfuscates by underproducing in the first stage, and the following holds in equilibrium:

1. In the first stage, producer j submits w_j^{L,CC^*} (i's belief is such that θ is set to 1) as his wholesale price and produces K_j^L regardless of type. Producer i submits $w_{i,2}^{CC^*}$ as his wholesale price and produces $D_{i,2}^{L,CC^*}$ units.
2. In the second stage, producer j with type K_j^L produces K_j^L and sets a wholesale price w_j^{L,CC^*}; producer j with type K_j^H produces K_j^H and sets a wholesale price w_j^{H,CC^*}, while producer i produces $D_{i,2}^{H,CC^*}$ and sets a wholesale price of $w_{i,2}^{CC^*}$.
3. Letting $q_{j,1}$ denote producer j's first-stage output, producer i's posterior belief is given by $\theta_2(q_{j,1} \leq K_j^L) = \theta$.
4. The retailer sets first and second-stage prices, respectively, to:

$$
\begin{aligned}
p_{j,1}^* &= \frac{[4-\gamma(1+2\gamma)]\,\alpha + (1-\gamma)\left[\gamma c_i - 4K_j^L\right]}{2(1-\gamma)(2-\gamma^2)} \\
p_{i,1}^* &= \frac{\left[3-2\gamma^2\right]\alpha + (1-\gamma)\left[c_i - 2\gamma K_j^L\right]}{2(1-\gamma)(2-\gamma^2)} \\
p_{i,2}^* &= \frac{\left[3-2\gamma^2\right]\alpha + (1-\gamma)\left[c_i - 2\gamma K_j^H\right] + 2\theta\gamma\,(1-\gamma)\left[K_j^H - K_j^L\right]}{2(1-\gamma)(2-\gamma^2)}
\end{aligned}
$$

If producer j is type K_j^L, then

$$p_{j,2}^* = \frac{[4-\gamma(1+2\gamma)]\,\alpha + (1-\gamma)\left[\gamma c_i - 4K_j^L\right] - 2\gamma^2\,(1-\theta)\,(1-\gamma)\left[K_j^H - K_j^L\right]}{2(1-\gamma)(2-\gamma^2)}$$

Whereas if producer j is type K_j^H, then:

$$p_{j,2}^* = \frac{[4-\gamma(1+2\gamma)]\,\alpha + (1-\gamma)\left[\gamma c_i - 4K_j^H\right] + 2\theta\gamma^2(1-\gamma)\left[K_j^H - K_j^L\right]}{2(1-\gamma)(2-\gamma^2)}$$

B.7 Proof of Proposition 6:

Recall that as part of the first-stage equilibrium, both types of producer j behave as type K_j^L, with producer i anticipating this pooling behavior. Table 3 shows the sales quantities and profits in Stages 1 and 2 for a low-type producer j, as well as total present-discounted profits under both the no obfuscation and obfuscation settings. We observe that under no obfuscation, producer i has a prior belief of θ, which gets updated to $\theta_2 = 1$ once he observes a first stage output of K_j^L, whereas under the obfuscation setting, producer i expects the pooling behavior and, thus, does not update his posterior beliefs.

We have $\Pi_{j,t}^L(K_j^L, \theta) = \Pi_{j,t}^{L,CC^*}(\theta) = \frac{\left[(2+\gamma)\alpha + \gamma c_i - (2-\gamma^2)c_j - 4K_j^L - 2\gamma^2(1-\theta)(K_j^H - K_j^L)\right]K_j^L}{2-\gamma^2}$, $t \in \{1,2\}$, given by (34), and $\Pi_{j,t}^L(K_j^L, \theta = 1) = \Pi_{j,t}^{L,CC^*}(\theta = 1) = \frac{\left[(2+\gamma)\alpha + \gamma c_i - (2-\gamma^2)c_j - 4K_j^L\right]K_j^L}{2-\gamma^2}$, $t \in \{1,2\}$.

From the above equalities, it follows that $\Pi_{j,1}^L(K_j^L, \theta = 1) > \Pi_{j,1}^L(K_j^L, \theta)$. Thus, a low-type producer j weakly prefers obfuscation to no obfuscation, and strictly so for $\delta < 1$.

Results for Producer i: In Table 4, we present the sales quantities and profits in Stages 1 and 2, as well as total discounted profits for producer i, under both the no obfuscation and obfuscation settings, when producer i faces competition from a high-type and a low-type producer j.

Table 3: Low-type Producer j's Sales Quantity and Profits in Stages 1 and 2, as well as Total Present-Discounted Profits, under both the No Obfuscation and Obfuscation Settings.

	Stage 1		Stage 2		Total Discounted Profits
	Sales	Profit	Sales	Profit	
No Obfuscation	K_j^L	$\Pi_{j,1}^L(K_j^L,\theta)$	K_j^L	$\Pi_{j,2}^L(K_j^L,\theta=1)$	$\Pi_{j,1}^L(K_j^L,\theta)+\delta\Pi_{j,2}^L(K_j^L,\theta=1)$
Obfuscation	K_j^L	$\Pi_{j,1}^L(K_j^L,\theta=1)$	K_j^L	$\Pi_{j,2}^L(K_j^L,\theta)$	$\Pi_{j,1}^L(K_j^L,\theta=1)+\delta\Pi_{j,2}^L(K_j^L,\theta)$

Table 4: Producer i's Sales Quantity and Profits in Stages 1 and 2, as well as Total Present-Discounted Profits, under both the "No Obfuscation" and "Obfuscation" Settings.

		Stage 1		Stage 2		Total Discounted Profits
		Sales	Profit	Sales	Profit	
High	No Obf.	$D_{i,1}^{H,CC^*}(\theta)$	$\Pi_{i,1}^{H,CC^*}(\theta)$	$D_{i,2}^{H,CC^*}(\theta=0)$	$\Pi_{i,2}^{H,CC^*}(\theta=0)$	$\Pi_{i,1}^{H,CC^*}(\theta)+\delta\Pi_{i,2}^{H,CC^*}(\theta=0)$
Type	Obf.	$D_{i,1}^{L,CC^*}(\theta=1)$	$\Pi_{i,1}^{L,CC^*}(\theta=1)$	$D_{i,2}^{H,CC^*}(\theta)$	$\Pi_{i,2}^{H,CC^*}(\theta)$	$\Pi_{i,1}^{L,CC^*}(\theta=1)+\delta\Pi_{i,2}^{H,CC^*}(\theta)$
Low	No Obf.	$D_{i,1}^{L,CC^*}(\theta)$	$\Pi_{i,1}^{L,CC^*}(\theta)$	$D_{i,2}^{L,CC^*}(\theta=1)$	$\Pi_{i,2}^{L,CC^*}(\theta=1)$	$\Pi_{i,1}^{L,CC^*}(\theta)+\delta\Pi_{i,2}^{L,CC^*}(\theta=1)$
Type	Obf.	$D_{i,1}^{L,CC^*}(\theta=1)$	$\Pi_{i,1}^{L,CC^*}(\theta=1)$	$D_{i,2}^{L,CC^*}(\theta)$	$\Pi_{i,2}^{L,CC^*}(\theta)$	$\Pi_{i,1}^{L,CC^*}(\theta=1)+\delta\Pi_{i,2}^{L,CC^*}(\theta)$

Here, $\Pi_{i,t}^{L,CC^*}(\theta) = \dfrac{\Big[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^H+2\theta\gamma(K_j^H-K_j^L)\Big]\cdot\Big[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^L+2(\gamma(1-\gamma^2)(1-\theta))\Big(K_j^H-K_j^L\Big)\Big]}{2(2-\gamma^2)^2}$, $t\in\{1,2\}$, is given by (33), and $\Pi_{i,t}^{L,CC^*}(\theta=1)=\frac{\left[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^L\right]^2}{2(2-\gamma^2)^2}$. Similarly, $\Pi_{i,t}^{H,CC^*}(\theta) = \dfrac{\Big[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^H-2\theta\gamma(1-\gamma^2)\Big(K_j^H-K_j^L\Big)\Big]\cdot\Big[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^H+2\theta\gamma(K_j^H-K_j^L)\Big]}{(2-\gamma^2)^2}$, is given by (31), and $\Pi_{i,t}^{H,CC^*}(\theta=0)=\dfrac{\left[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^H\right]^2}{2\left(2-\gamma^2\right)^2}$, $t\in\{1,2\}$.

Producer i's Present-Discounted Profit when Competing against a Low-Type Producer j: We note that $\Pi_{i,t}^{L,CC^*}(\theta=1)-\Pi_{i,t}^{L,CC^*}(\theta)$

$$= \frac{(1-\theta)\gamma^2(K_j^H-K_j^L)\left[\gamma(1+\gamma)\left(\alpha-(1-\gamma)c_i\right)-2\gamma^2K_j^L+2(1-\gamma^2)(1-\theta)(K_j^H-K_j^L)\right]}{[2-\gamma^2]^2} > 0$$

This follows since $w_{i,t}^{L,CC^*}\geq c_i$ (as a participation constraint) gives $2\gamma^2K_j^L\leq\gamma(1+\gamma)\left(\alpha-(1-\gamma)c_i\right)$. Thus, when competing against a low-type producer j, for any δ, producer i's present-discounted profit under obfuscation exceeds his present-discounted profit under no obfuscation.

Producer i's Present-Discounted Profits when Competing against a High-Type Producer j: It follows readily that $\Pi_{i,1}^{L,CC^*}(\theta=1)>\Pi_{i,1}^{H,CC^*}(\theta)$, i.e., the first stage profit under obfuscation exceeds the first stage profit with no obfuscation, and that $\Pi_{i,2}^{H,CC^*}(\theta)>\Pi_{i,2}^{H,CC^*}(\theta=0)$, i.e., producer i's second-stage profit under obfuscation exceeds his second-stage profit with no obfuscation. Hence, producer i always prefers the obfuscation setting to no obfuscation.

B.8 Proof of Proposition 7

Table 5 shows the wholesale prices submitted by producer i and a low-type producer j, the resulting sales quantities, as well as profits for the retailer in both game stages, under the no obfuscation and obfuscation settings.

Table 5: Wholesale Prices offered by Producer i and a Low-Type Producer j, the Resulting Sales Quantities, and Retailer Profits in Stage 1 and 2, under the No Obfuscation and Obfuscation Settings.

		w_i	D_i	w_j	D_j	Profits
Stage 1	No Obfuscation	$w_{i,1}^{L,CC^*}(\theta)$	$D_{i,1}^{L,CC^*}(\theta)$	$w_{j,1}^{L,CC^*}(\theta)$	K_j^L	$\Pi_{R,1}^L(K_j^L,\theta)$
	Obfuscation	$w_{i,1}^{L,CC^*}(\theta=1)$	$D_{i,1}^{L,CC^*}(\theta=1)$	$w_{j,1}^{L,CC^*}(\theta=1)$	K_j^L	$\Pi_{R,1}^L(K_j^L,\theta=1)$
Stage 2	No Obfuscation	$w_{i,2}^{L,CC^*}(\theta=1)$	$D_{i,2}^{L,CC^*}(\theta=1)$	$w_{j,2}^{L,CC^*}(\theta=1)$	K_j^L	$\Pi_{R,2}^L(K_j^L,\theta=1)$
	Obfuscation	$w_{i,2}^{L,CC^*}(\theta)$	$D_{i,2}^{L,CC^*}(\theta)$	$w_{j,2}^{L,CC^*}(\theta)$	K_j^L	$\Pi_{R,2}^L(K_j^L,\theta)$

We have $w_{i,t}^{L,CC^*}(\theta) = \frac{(1+\gamma)\alpha+c_i-2\gamma K_j^H+2\theta\gamma(K_j^H-K_j^L)}{2-\gamma^2}$, $w_{i,t}^{L,CC^*}(\theta=1) = \frac{(1+\gamma)\alpha+c_i-2\gamma K_j^H+2\gamma(K_j^H-K_j^L)}{2-\gamma^2}$, $w_{j,t}^{L,CC^*}(\theta) = \frac{(2+\gamma)\alpha+\gamma c_i-4K_j^L-2\gamma^2(1-\theta)(K_j^H-K_j^L)}{2-\gamma^2}$, $w_{j,t}^{L,CC^*}(\theta=1) = \frac{(2+\gamma)\alpha+\gamma c_i-4K_j^L}{2-\gamma^2}$, $D_{i,t}^{L,CC^*}(\theta) = \frac{(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^L+2\gamma\left[(1-\gamma^2)(1-\theta)\right](K_j^H-K_j^L)}{2(2-\gamma^2)}$ and $D_{i,t}^{L,CC^*}(\theta=1) = \frac{(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^L}{2(2-\gamma^2)}$, $t\in\{1,2\}$.

It can be readily shown that $w_{j,1}^{L,CC^*}(\theta=1) > w_{j,1}^{L,CC^*}(\theta)$ and $w_{i,1}^{L,CC^*}(\theta=1) > w_{i,1}^{L,CC^*}(\theta)$. Since producers are availing the same capacity to the retailer under both the obfuscation and no obfuscation cases, and since both wholesale prices are higher under obfuscation, it follows that the retailer is strictly worse off in the first stage given the possibility of producer j obfuscating; that is, $\Pi_{R,1}^L(K_j^L,\theta) > \Pi_{R,1}^L(K_j^L,\theta=1)$.

Comparing the case with no obfuscation to the case with obfuscation, the retailer's present-discounted profits are given by $\Pi_{R,1}^L(K_j^L,\theta)+\delta\Pi_{R,2}^L(K_j^L,\theta=1)$ vs. $\Pi_{R,1}^L(K_j^L,\theta=1)+\delta\Pi_{R,2}^L(K_j^L,\theta)$. Given $\Pi_{R,1}^L(K_j^L,\theta) > \Pi_{R,1}^L(K_j^L,\theta=1)$, it follows that the retailer is better off under no obfuscation, and strictly so for $\delta<1$.

Similarly, Table 6 shows the wholesale prices submitted by producer i and a high-type producer j, the resulting sales quantities, as well as profits for retailer, under the no obfuscation and obfuscation settings.

Table 6: Wholesale Prices offered by Producer i and a High-Type Producer j, the Resulting Sales Quantities, and Retailer Profits in Stage 1 and 2, under the No Obfuscation and Obfuscation Settings.

		w_i	D_i	w_j	D_j	Profits
Stage 1	No Obfuscation	$w_{i,1}^{H,CC^*}(\theta)$	$D_{i,1}^{H,CC^*}(\theta)$	$w_{j,1}^{H,CC^*}(\theta)$	K_j^H	$\Pi_{R,1}^H(K_j^H,\theta)$
	Obfuscation	$w_{i,1}^{L,CC^*}(\theta=1)$	$D_{i,1}^{L,CC^*}(\theta=1)$	$w_{j,1}^{L,CC^*}(\theta=1)$	K_j^L	$\Pi_{R,1}^L(K_j^L,\theta=1)$
Stage 2	No Obfuscation	$w_{i,2}^{H,CC^*}(\theta=0)$	$D_{i,2}^{H,CC^*}(\theta=0)$	$w_{j,2}^{H,CC^*}(\theta=0)$	K_j^H	$\Pi_{R,2}^H(K_j^L,\theta=0)$
	Obfuscation	$w_{i,2}^{H,CC^*}(\theta)$	$D_{i,2}^{H,CC^*}(\theta)$	$w_{j,2}^{H,CC^*}(\theta)$	K_j^H	$\Pi_{R,2}^H(K_j^H,\theta)$

We have $w_{i,t}^{H,CC^*}(\theta) = \frac{(1+\gamma)\alpha+c_i-2\gamma K_j^H+2\theta\gamma(K_j^H-K_j^L)}{2-\gamma^2}$, $w_{i,2}^{H,CC^*}(\theta=0) = \frac{(1+\gamma)\alpha+c_i-2\gamma K_j^H}{2-\gamma^2}$,

$w_{j,t}^{H,CC^*}(\theta) = \frac{(2+\gamma)\alpha+\gamma c_i-4K_j^H+2\theta\gamma^2(K_j^H-K_j^L)}{2-\gamma^2}$, $w_{j,2}^{H,CC^*}(\theta=0) = \frac{(2+\gamma)\alpha+\gamma c_i-4K_j^H}{2-\gamma^2}$. $w_{i,1}^{L,CC^*}(\theta = 1) = \frac{(1+\gamma)\alpha+c_i-2\gamma K_j^H+2\gamma(K_j^H-K_j^L)}{2-\gamma^2}$, and $w_{j,1}^{L,CC^*}(\theta = 1) = \frac{(2+\gamma)\alpha+\gamma c_i-4K_j^L}{2-\gamma^2}$, where $D_{i,t}^{H,CC^*}(\theta) = \frac{(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^H-2\theta\gamma(1-\gamma^2)(K_j^H-K_j^L)}{2(2-\gamma^2)}$, $D_{i,2}^{H,CC^*}(\theta=0) = \frac{(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^H}{2(2-\gamma^2)}$, and $D_{i,1}^{L,CC^*}(\theta=1) = \frac{(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^L}{2(2-\gamma^2)}$, $t \in \{1,2\}$.

We note that $w_{i,1}^{L,CC^*}(\theta = 1) > w_{i,t}^{H,CC^*}(\theta)$, $w_{j,1}^{L,CC^*}(\theta = 1) > w_{j,t}^{H,CC^*}(\theta)$, and $w_{i,1}^{L,CC^*}(\theta = 1) > w_{i,1}^{L,CC^*}(\theta)$. This implies that the retailer faces higher wholesale prices, while selling fewer units of product j as a result of j underproducing in the first stage. It follows that the retailer is strictly worse off in the first stage given the possibility of producer j obfuscating; that is, $\Pi_{R,1}^H(K_j^L,\theta) > \Pi_{R,1}^L(K_j^L,\theta=1)$. An analogous comparison in the second stage reveals that the retailer's present-discounted expected profits under no obfuscation are higher than his profits under the obfuscation setting, that is, $\Pi_{R,1}^H(K_j^H,\theta) + \delta\Pi_{R,2}^H(K_j^L,\theta=0) > \Pi_{R,1}^L(K_j^L,\theta=1) + \delta\Pi_{R,2}^H(K_j^H,\theta)$. Figure 6 depicts the retailer's total discounted profits, given both a low-type (left) and a high-type (right) producer j. Since the retailer is made worse off as a result of obfuscation regardless of j's realized

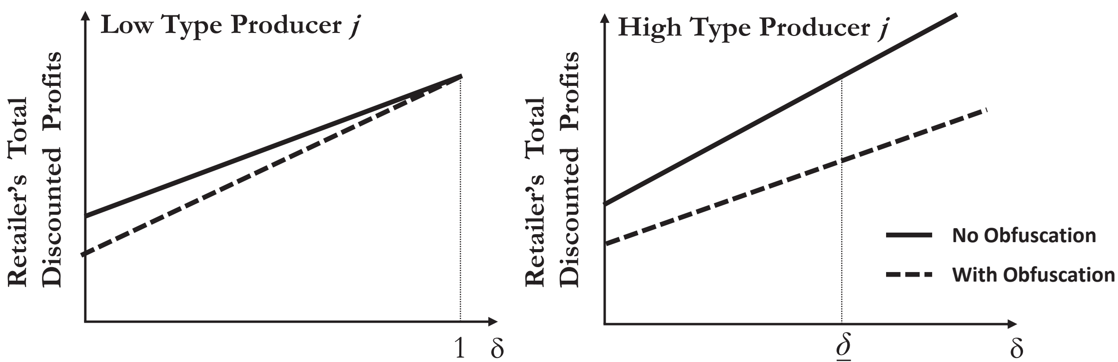

Figure 6: Total Discounted Profits for the Retailer, Conditional on Producer j's Type.

type, the retailer's ex-ante expected profit, which is a convex combination of the two, is also lower relative to the case with no obfuscation.

Channel Profits: The expression for present-discounted channel profits can be derived from the individual player profits above. With a high-type producer j, under the no obfuscation setting, we have

$$\Pi_{Channel,\text{No OBF}}^{CC^*} = \overbrace{\Pi_{i,1}^{H,CC^*}(\theta) + \Pi_{j,1}^{H,CC^*}(\theta) + \Pi_{R,1}^{H,CC^*}(\theta)}^{\text{Profits in the First Stage}} + \underbrace{\delta\left[\Pi_{i,2}^{H,CC^*}(\theta=0) + \Pi_{j,2}^{H,CC^*}(\theta=0) + \Pi_{R,2}^{H,CC^*}(\theta=0)\right]}_{\text{Discounted Second-Stage Profits}}$$

Under Obfuscation, the discounted channel profits are:

$$\Pi_{Channel,\text{OBF}}^{CC^*} = \overbrace{\Pi_{i,1}^{L,CC^*}(\theta=1) + \Pi_{j,1}^{L,CC^*}(\theta=1) + \Pi_{R,1}^{L,CC^*}(\theta=1)}^{\text{Profits in the First Stage}}$$

$$+\underbrace{\delta\left[\Pi_{i,2}^{H,CC^*}(\theta)+\Pi_{j,2}^{H,CC^*}(\theta)+\Pi_{R,2}^{H,CC^*}(\theta)\right]}_{\text{Discounted Second-Stage Profits}}$$

Substituting the relevant expressions in calculating the difference, leads to:
$\Pi^{CC^*}_{Channel,\text{No OBF}} - \Pi^{CC^*}_{Channel,\text{OBF}}$

$$\begin{aligned} =\ & 4\gamma(1-\gamma^2)\left[\alpha+\gamma c_j-c_i\right]\left(K_j^H-K_j^L\right)\\ & -4\theta\gamma(1-\gamma^2)\left[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^H+\theta\gamma(K_j^H-K_j^L)\right]\left(K_j^H-K_j^L\right)\\ & +4(4-3\gamma^2)\left[\alpha+\gamma c_i-c_j-(K_j^H+K_j^L)\right]\left(K_j^H-K_j^L\right)\\ & +\delta\left[4\theta\gamma(1-\gamma^2)\left[(1+\gamma)\alpha-(1-\gamma^2)c_i-2\gamma K_j^H+\theta\gamma(K_j^H-K_j^L)\right]\right]\left(K_j^H-K_j^L\right)\\ & -(1-\gamma^2)(c_i^2-c_i) \end{aligned} \tag{43}$$

A sufficient condition for the above to be greater than or equal to zero is given by:

$$K_j^L \leq \frac{\alpha-c_j+\gamma c_i}{2}-\frac{\sqrt{\begin{array}{l}(4-3\gamma^2)^2\left[(\alpha-c_j)^2-4(\alpha-c_j-K_j^H)K_j^H\right]\\ +\gamma\left[2\alpha-2c_j+\gamma c_i-4K_j^H\right]c_i+(4-3\gamma^2)\left[(1-\gamma^2)(c_i^2-c_i)\right]\end{array}}}{2(4-3\gamma^2)}\equiv \bar{K}_j^L$$

It can also be readily seen from (43) that the *necessary* condition is weaker as either the prior θ decreases and/or the discount factor δ increases.

C Results Related to Section 5

C.1 Proof of Proposition 8

The upper bound on c_V is given by the retailer's profit gain relative to the no-obfuscation case. In the case where producer j is constrained in equilibrium in the no-obfuscation setting, we have

$$\begin{aligned} \bar{c}_V^C &= \Pi_{R,2}^{C^*}-\Pi_{R,2}^{CC^*}\\ &= \frac{\theta\gamma}{(2-\gamma^2)^2}\left[\begin{array}{c}\gamma(1-\gamma^2)(2-\theta)\left(K_j^H\right)^2+(1+\gamma)(\alpha-c_i+\gamma c_i)\left(K_j^H-K_j^L\right)\\ -2\gamma(1-\gamma^2)(1-\theta)K_j^HK_j^L-\theta\gamma(1-\gamma^2)\left(K_j^L\right)^2\end{array}\right]\\ &= \frac{\theta\gamma}{(2-\gamma^2)^2}\left[\begin{array}{l}(1+\gamma)(\alpha-c_i+\gamma c_i)\left(K_j^H-K_j^L\right)+2\gamma(1-\gamma^2)\left(\left(K_j^H\right)^2-K_j^HK_j^L\right)\\ +\theta\gamma(1-\gamma^2)\left[2K_j^HK_j^L-\left(\left(K_j^H\right)^2+{K_j^L}^2\right)\right]\end{array}\right]\geq 0 \end{aligned}$$

as $\alpha-c_i+\gamma c_i\geq 0$ is demand for either product if sold at producer i's marginal production cost, and because K_j^L is bounded above by K_j^H, the above is guaranteed to be non-negative.

To show the second portion of the proposition, we note that

$$\frac{\partial c_V^C}{\partial K_j^L}=\underbrace{-\frac{\theta\gamma}{[2-\gamma^2]^2}}_{\leq 0}\left[\ (1+\gamma)\alpha-(1-\gamma^2)c_i+2\gamma(1-\theta)(1-\gamma^2)K_j^H+2\gamma\theta(1-\gamma^2)K_j^L\ \right]\leq 0$$

The term in square brackets is non-negative for $\alpha\geq\frac{1}{1-\gamma^2}c_i$.

In the case where producer j is unconstrained in equilibrium in the no-obfuscation case, we have

$$\bar{c}_V^U = \Pi_{R,2}^{U^*} - \Pi_{R,2}^{UC^*} \quad = \quad \frac{\begin{array}{l}\theta\gamma \cdot \left[\alpha(2+\gamma) + \gamma c_i - (2-\gamma^2)c_j - 2(4-\gamma^2)K_j^L\right] \cdot \\ \left[\begin{array}{l} (2+\gamma)\left(16 + 4\gamma(2-\theta) - 4\gamma^2(1+\theta) - \gamma^3(2-\theta)\right)\alpha \\ -\left[(8-\gamma^2(2+\theta))(4-3\gamma^2)\right]c_i + \left[(2\gamma^2(4-5\theta) - \gamma^4(2-\theta) + 8\theta)\right]\gamma c_j \\ \quad +2\theta\gamma(4-\gamma^2)(4-3\gamma^2)K_j^L \end{array}\right]\end{array}}{4(4-\gamma^2)^2\left[4-\gamma^2(1+\theta)\right]^2}$$

as an upper bound on the amount the retailer is willing to spend to verify producer j's capacity.

Similarly, for the second portion of the proposition, we have

$$\frac{\partial c_V^U}{\partial K_j^L} = \underbrace{\frac{\theta\gamma}{4(4-\gamma^2)[4-\gamma^2(1+\theta)]^2}}_{\geq 0} \left[\begin{array}{l} -8(2+\gamma)^2(2-\gamma)^2\left[2(1+\gamma)\alpha\right] + 8(2+\gamma)^2(2-\gamma)^2\theta\gamma\left[\alpha - c_j\right] \\ -4(2+\gamma)^2(2-\gamma)^2(1-2\theta)\gamma^2\left[\alpha + \gamma c_j\right] \\ -4(4-3\gamma^2)(2+\gamma)^2(2-\gamma)^2\left[2\theta\gamma K_j^L - c_i\right] \end{array}\right] \leq 0$$

C.2 Proof of Proposition 9:

Let us consider the CC case; the UC case is solved analogously. Suppose the retailer commits in advance to order $q \in [0, K_j^L]$ units from a producer j who sets a wholesale price less than $w_{j,1}^{H,CC}(w_{i,1}^{CC}(q)) = \alpha + \gamma w_{i,1}^{CC}(q) - 2K_j^H$, where j's wholesale price is set to maximize his profit. Producer j's deviation profit, that is, his profit from producing q when his capacity is K_j^H, is given by:

$$\Pi_{j,1}^H(q,\theta) = \left[\left(\alpha + \gamma\left[\frac{(1+\gamma)\alpha + c_i - 2\gamma K_j^H + 2\theta\gamma\left(K_j^H - q\right)}{2-\gamma^2}\right] - 2q\right) - c_j\right] \cdot q$$

To successfully deter obfuscation, the retailer must choose a quantity q such that $\Pi_{j,1}^H(K_j^H,\theta) - \Pi_{j,1}^H(q,\theta) - \delta\left[\Pi_{j,2}^H(K_j^H,1) - \Pi_{j,2}^H(K_j^H,0)\right] \geq 0$; that is, an output level such that producer j's expected profit when producing according to his true capacity exceeds his deviation profit including any second-stage gains from obfuscation. To show that a range of such quantities always exists, consider the case where $\delta = 1$ and the retailer sets $q = 0$. We need:

$$\Pi_{j,1}^H(K_j^H,\theta) - \Pi_{j,1}^H(0,\theta) - \delta\left[\Pi_{j,2}^H(K_j^H,1) - \Pi_{j,2}^H(K_j^H,0)\right]$$

$$= \quad \frac{1}{2-\gamma^2}\left[(2+\gamma)\alpha + \gamma c_i - (2-\gamma^2)c_j - 4K_j^H - 2\gamma^2(\delta-\theta)(K_j^H - K_j^L)\right] \geq 0,$$

which is indeed satisfied from producer j's participation constraint (wholesale price exceeds marginal cost). By continuity, there exists a quantity level $q^R(\delta = 1) > 0$ such that for all $q \in [0, q^R]$, producer j will not underproduce. Since second-stage gains from obfuscation are highest under $\delta = 1$, it follows that there exists $q^R(\delta)$ for any $\delta \in [0,1)$.

Consider the retailer's policy of deterring obfuscation by ordering a quantity $q_R < K_j^L$ from an apparent low-capacity producer j. Then by deterring producer j from underproducing in the first stage, the retailer's expected profit changes by

$$\underbrace{(1-\theta)\left[\Pi_{R,1}^{H,CC}(\theta) - \Pi_{R,1}^{L,CC}(\theta=1) + \delta(\Pi_{R,2}^{H,CC}(\theta=0) - \Pi_{R,2}^{H,CC}(\theta))\right]}_{\text{Retailer's gain from deterring obfuscation given a high-type producer } j} \quad -$$

$$\underbrace{\theta\left[\Pi_{R,1}^{q_R,CC}(\theta) - \Pi_{R,1}^{L,CC}(\theta=1) + \delta(\Pi_{R,1}^{L,CC}(\theta=1) - \Pi_{R,2}^{L,CC}(\theta))\right]}_{\text{Retailer's net gain/loss from countering obfuscation by ordering } q_R<K_j^L \text{ units in the first stage}}$$

Since the first bracketed term is positive by Proposition 7, it follows that there exists a threshold on the prior belief regarding producer j's type, $\bar{\theta}(\delta)$, such that for all $\theta \in [0, \bar{\theta}(\delta)]$ the retailer is better off committing ex ante to purchasing q_R units from an apparent low-capacity producer j in the first stage; that is, the retailer is better off committing to deter obfuscation through output reduction.

References

[1] G. Allon and A. Bassamboo. Buying from the babbling retailer? The impact of availability information on customer behavior. *Management Science*, 57(4):713–726, 2011.

[2] K. Anand and M. Goyal. Strategic information management under leakage in a supply chain. *Management Science*, 55(3):438–452, 2009.

[3] J.B. Baker. Two Sherman Act section 1 dilemmas: Parallel pricing, the oligopoly problem, and contemporary economic theory. *Antitrust Bulletin*, 38:143, 1993.

[4] S. Benjaafar, Cooper W. L., and S. Mardan. Production-inventory systems with imperfect advance demand information and updating. *Naval Research Logistics*, 38:88–106, 1993.

[5] S. Borenstein, J.B. Bushnell, and F.A. Wolak. Measuring market inefficiencies in California's restructured wholesale electricity market. *American Economic Review*, 92(5):1376–1405, 2002.

[6] G. Cachon and M. Lariviere. Capacity choice and allocation: Strategic behavior and supply chain performance. *Management Science*, 45(8):1091–1108, 1999.

[7] L.M.A. Chan, Z.J.M. Shen, D. Simchi-Levi, and J. Swann. Coordination of pricing and inventory decisions: A survey and classification. In *Supply Chain Analysis in the eBusiness Era*, pages 335–392. Kluwer Academic Publishers, Boston, MA., 2004.

[8] S. Cho. The optimal composition of influenza vaccines subject to random production yields. *Manufacturing & Service Operations Management*, 12(2):256–277, 2010.

[9] L.Y. Chu, N. Shamir, and H. Shin. Strategic communication for capacity alignment with pricing in a supply chain. *Working Paper, Marshall School of Business, University of Southern California, Los Angeles CA*, 2012. Available at `SSRN:http://ssrn.com/abstract=1944668`.

[10] W. Chu. Demand signalling and screening in channels of distribution. *Marketing Science*, 11(4):327–347, 1992.

[11] V. P. Crawford and J. Sobel. Strategic information transmission. *Econometrica*, 50(6):1431–1451, 1982.

[12] P. Desai and K. Srinivasan. Demand signalling under unobservable effort in franchising: Linear and non-linear contracts. *Management Science*, 41(10):1608–1623, 1995.

[13] W. Elmaghraby and P. Keskinocak. Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. *Management Science*, 49(10):1287–1309, 2003.

[14] M. E. Ferguson, G.A. DeCroix, and P. H. Zipkin. Commitment decisions with partial information updating. *Naval Research Logistics*, 52(8):780–795, 2005.

[15] Y. Gerchak, E. Khmelnitsky, and L.W. Robinson. Untruthful probabilistic demand forecasts in vendor-managed revenue-sharing contracts: Coordinating the chain. *Naval Research Logistics*, 54(7):742–749, 2007.

[16] M. Gümüs, S. Ray, and H. Gurnani. Supply side story: Risks, guarantees, and information asymmetry. *Management Science*, 58(9):1694–1714, 2012.

[17] A. Y. Ha. Supplier-buyer contracting: Asymmetric cost information and cutoff level policy for buyer participation. *Naval Research Logistics*, 48(1):41–64, 2001.

[18] Ö. İşlegen and E.L. Plambeck. Capacity leadership. *Working Paper, Kellogg School of Management, Northwestern University, Evanston IL*, 2011. Available at: `http://www.kellogg.northwestern.edu/~/media/Files/Faculty/Research/ArticlesBookChaptersWorkingPapers/Early_Capacity_v40-Names.ashx`.

[19] P.L. Joskow and E. Kahn. A quantitative analysis of pricing behavior in California's wholesale electricity market during summer 2000: The final word. *Working Paper, Massachusetts Institute of Technology, Cambridge, MA*, 2002. Available at `http://econ-www.mit.edu/faculty/pjoskow/files/Joskow-K.pdf`.

[20] H. Konishi and M. Unver. Games of capacity manipulation in hospital-intern markets. *Social Choice and Welfare*, 27(1):3–24, 2006.

[21] M.A. Lariviere and V. Padmanabhan. Slotting allowances and new product introductions. *Marketing Science*, 16(2):112–128, 1997.

[22] V. Martinez-de Albeniz and K. Talluri. Dynamic price competition with fixed capacities. *Management Science*, 57(6):1078–1093, 2011.

[23] McCullough Research Report. May and October 2012 gasoline price spikes on the West coast. *McCullough Research Report*, 2012. Available at `http://www.mresearch.com/pdfs/489.pdf`.

[24] H. Nazerzadeh and G. Perakis. Menu pricing competition with private capacity constraints. *Working Paper, Marshall School of Business, University of Southern California, Los Angeles CA*, 2012.

[25] Ö. Özer and W. Wei. Strategic commitments for an optimal capacity decision under asymmetric forecast information. *Management Science*, 52(8):1238–1257, 2006.

[26] Ö. Özer, Y. Zheng, and K.-Y. Chen. Trust in forecast information sharing. *Management Science*, 57(6):1111–1137, 2011.

[27] E. L. Porteus, H. Shin, and T. I. Tunca. Feasting on leftovers: Strategic use of shortages in price competition among differentiated products. *Manufacturing & Service Operations Management*, 12(1):140–161, 2010.

[28] S. L. Puller. Pricing and firm conduct in California's deregulated electricity market. *Review of Economics and Statistics*, 89(1):75–87, 2007.

[29] J. Riley. Silver signals: Twenty-five years of screening and signaling. *Journal of Economic Literature*, 39(2):432–478, 2001.

[30] M. Spence. Job market signaling. *Quarterly Journal of Economics*, 87(3):355–374, 1973.

[31] A. Stock and S. Balachander. The making of a "hot product": A signaling explanation of marketers? scarcity strategy. *Management Science*, 51(8):355–374, 2005.

[32] X. Su. Optimal pricing with speculators and strategic consumers. *Management Science*, 56(1):25–40, 2010.

[33] A. C. Tellidou and A.G. Bakirtzis. Agent-based analysis of capacity withholding and tacit collusion in electricity markets. *IEEE Transactions on Power Systems*, 22(4):1735–1742, 2007.

[34] B. Tomlin. Impact of supply learning when suppliers are unreliable. *Manufacturing & Service Operations Management*, 11(2):192–209, 2009.

[35] F.A. Wolak and R.H. Patrick. The impact of market rules and market structure on the price determination process in the England and Wales electricity market. *NBER Working Paper No. 8248*, 2001. Available at: http://www.nber.org/papers/w8248.

[36] O. Wu and V. Babich. Unit-contingent power purchase agreement and asymmetric information about plant outage. *Manufacturing & Service Operations Management*, 14(2):245–261, 2012.

[37] C.A. Yano and E.J. Durango-Cohen. Impact of capacity on pricing decisions in supply chains with competing store and national brands. *Working Paper, Haas School of Business, University of California–Berkeley*, 2007.

[38] C.A. Yano, E.J. Durango-Cohen, and L. Wagman. Outsourcing in place: Should a retailer sell its store-brand factory? *Working Paper, Haas School of Business, University of California–Berkeley*, 2011.

[39] C.A. Yano and S.M. Gilbert. Coordinated pricing and production/procurement decisions: A review. In A. Chakravarty and J. Eliashberg, editors, *Managing Business Interfaces: Marketing, Engineering and Manufacturing Perspectives*, chapter 3, pages 61–103. Kluwer Academic Publishers, Boston, MA., 2003.

[40] Qing Ye, Izak Duenyas, and Roman Kapuscinski. Should competing firms reveal their capacity? *Naval Research Logistics*, 60(1):64–86, 2013.