Outsourcing in Place: Should a Retailer Sell its Store-Brand Factory?

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Abstract

Several major grocery chains in the US own factories that produce some of their store-brand products. Historically, these store-brand products have been the low-price, lower-quality alternatives to higher-priced national brands, but the quality and consumer acceptance of store brands have increased markedly in recent years. While demand for store-brand products is higher, managing the associated factories can be costly for retailers, leading some to consider selling the factories to third parties.

We study the impact of selling a retailer's existing capacity-limited factory to a third party when a store-brand product competes with a similar national-brand product. We examine the equilibrium dynamics between two external suppliers and show how the outcome changes with respect to prices, capacity limitations, the distribution of profits, and the sequencing of pricing decisions. Among other things, we show that, surprisingly, the national brand's equilibrium wholesale price may fall when the factory is sold. We also show that the retailer may be strictly better off if he sells the factory, with these benefits being above and beyond any savings in fixed ownership and operating costs. Taken together, these results imply that when the store-brand factory has tight capacity, the adverse effects due to double marginalization on the store-brand product from selling the factory to a third party may be partially or fully offset by a reduction in the national brand's wholesale price.

Keywords: Store brands; private label; capacity constraints; pricing; outsourcing.

1 Introduction

Our work on this problem was motivated by a question that a Vice President of a large grocery chain posed to one of the authors. He said, "We are in the grocery retailing business, not in the manufacturing business. But we own factories that produce many of our private-label (mostly food) products. Would it make sense to sell these factories to firms that would serve as suppliers?" In addressing these issues, there are, of course, questions about whether greater focus would benefit the grocery chain in ways that might be difficult to forecast or analyze. In this paper, we focus on the direct effects of ownership restructuring on the competitive landscape in equilibrium.

Internal production of some store-brand food products is prevalent among large grocers (Connor et al., 1996). The grocery chain Kroger has one of the largest networks of private label manufacturing in North America. Over three dozen US plants (wholly owned or used with operating agreements) produce about 40% of Kroger's private label products (TheKrogerCo.com, 2014). Safeway, the second-largest US supermarket chain, operates more than 30 food processing plants in North America for products such as milk, bakery goods, soft drinks, and pet food (Safeway, 2013). The supermarket chain HEB operates several large manufacturing facilities in Texas, including milk and bread processing plants, and produces many other own-brand products, including ice cream, snacks, and ready-cooked meats and meals (Forbes.com, 2012). The grocer Publix owns 10 Publix-brand manufacturing facilities that produce its dairy, delicatessen, bakery, and other food products (Publix.com, 2014). A&P supermarkets pioneered the use of store brands, and its Eight O'Clock (sold off in 2003), Red Circle, and Bokar coffees were as notable as some national brands (A&P, 2014).

There is a trend toward financial divestiture of factories while retaining them as suppliers. One recent example in the grocery sector is Safeway's sale of its Joplin, Missouri manufacturing facility to Annie's, which will continue to produce Safeway's products in the same plant under a long-term contract (Annie's, Inc., 2013). Another recent example of a sale of a store-brand factory, also in the grocery industry, is the Canadian grocery chain Sobeys, who sold four dairy processing facilities to the dairy cooperative Agropur which will continue to manufacture for Sobeys (Canadian Press, 2014). Cohen (2013) provides an empirical analysis of the effect of yet another divestiture of store-brand milk production by a grocery chain (with a preferred-supplier relationship post-divestiture) in the northeastern part of the US. This phenomenon might be referred to as "outsourcing in place"—outsourcing at exactly the same location.

At the grocer that motivated our research, the store-brand factories, some built long ago, are approaching full capacity utilization because the grocer has added new stores and customers have become more receptive to store-brand products. As one example from a different grocery chain, Kroger reports that its 15 dairies and three ice cream plants operate at nearly full capacity, and that its private label milk is the "national brand" for most customers (TheKrogerCo.com, 2013). Many large grocery chains own food-processing facilities but within the past decade, third-party private-label manufacturing has grown dramatically.

In our motivating examples, there would a symbiotic relationship between the retailer and the third-party producer of the store brand if the ownership of the store-brand factory were transferred. The retailer is large enough that it would have essentially no alternative sources of production capacity for products such as milk, and few sources of capacity for other products, considering the massive volumes that the retailer requires. The third-party producer, likewise, would have few alternate customers for the massive amount of capacity that exists in the factories. This is due to limited alternative markets for the products that the factories produce, and the high cost of transporting over long distances perishable products such as fresh milk and bulky goods such as bakery items. Thus, neither party could easily walk away from the arrangement. And indeed, they may not enter into a "spin-off" arrangement if it were not in the best interests of both parties.

The retailer, or a third-party producer of the store brand, could, in principle, add capacity, but there are enormous economies of scale in food processing, and the vast majority of store-brand products that are produced in–house are food products. Unless the retailer plans to drastically increase its number of retail stores or the volume of products to be produced in–house, the investment in an additional factory, or an expansion in an existing factory is difficult to justify. Securing capacity at a factory owned by a third-party manufacturer may be a possibility, but that capacity may not be nearby (which is quite important for perishable products such as milk, bulky products such as bread, and heavy products such as canned fruits and vegetables).

We model a situation with a single retailer, one national-brand product, and one store-brand product that competes directly with the latter. To capture the fact that the products are perceived differently by customers and are not completely substitutable, we consider a model with differentiated products. The national brand offers a wholesale price being cognizant of the competition from the supplier of the store-brand product, whether that product is produced "in-house" or by a third party. Throughout the paper, we assume that the third party is not the national brand manufacturer; in the latter case, the channel structure would be quite different. We leave this case for future research. We consider two different mechanisms by which the wholesale price of the store brand is determined when it is produced by a third party: (i) selected by the third-party producer; and (ii) negotiated between the third-party producer and the retailer. The retailer sets retail prices for the two products, seeking to maximize his total profit. For case (i), we consider two sub-cases: one in which the national-brand manufacturer is the Stackelberg leader, which is appropriate when the national-brand manufacturer is a dominant player in the market and the third-party producer of the store brand product is relatively weak. The second sub-case involves a Nash game between the national brand and third-party supplier, which is appropriate when the third-party producer wields market power similar to that of the national-brand manufacturer. Although firms that specialize in producing store brands are not well-known because their firm names often do not even appear on the items that they produce, some of these firms have become quite large and have consolidated considerable market power in recent years. Some national-brand manufacturers produce store-brand products in different product categories than their own products, and these manufacturers often have substantial market power, as well. In these cases, the producer of the store-brand product is powerful enough that Nash competition between the national-brand manufacturer and the store-brand producer is a more accurate representation of reality than the national-brand manufacturer Stackelberg representation.

In other situations, the retailer may have more market power than the national- and store-brand manufacturers. Obtaining equilibria for such settings, however, can be quite complex. In particular, unless there is a mechanism to ensure that the retailer commits to decisions regarding retail margins on the two products, there is no guarantee of a stable equilibrium. In view of these challenges, we leave such scenarios for future research.

We explicitly account for the capacity limits of the store-brand factory. To capture the fact that the national-brand manufacturer has some latitude in allocating his capacity among many retailers, we assume that the national-brand manufacturer does not have strict capacity limits. Thus, if the national-brand manufacturer is operating near his production capacity limit, the economic effects of capacity factors can be captured by adding the national-brand manufacturer's opportunity cost of capacity to his variable cost of production when performing the equilibrium analysis. We also note that in a separate analysis which is not presented here, we have derived results for the case where the national brand manufacturer has allocated a fixed capacity to the retailer in question. The qualitative findings regarding conditions in which the retailer should sell the store-brand factory are similar, although the details of the equilibrium differ.

We also assume that shelf space constraints do not limit sales of either product. In practice, if the shelves are restocked often enough, shelf-space limitations will not severely restrict the quantities that the retailer is able to sell. Alternatively, one can adjust the demand functions to account for the aggregate effects of the retailer's restocking policy on the ultimate sales of the products.

Our primary aim is to answer the question of whether there are conditions under which the retailer can benefit from selling his store-brand factory. Our key findings are as follows. First, we find that there exists a range of capacity levels of the store-brand factory over which the national brand's equilibrium wholesale price is *smaller* when the retailer sells his factory than it is when the store-brand product is produced internally. Thus, despite the introduction of double marginalization on the store-brand product, the national brand may offer a lower price. This type of shift in the national brand's equilibrium price tends to occur when the capacity of the store-brand factory is relatively low, and although it is more profitable for the retailer to produce his store-brand in-house than not to offer it at all, at such capacity levels, the retailer may be even better off selling the store-brand factory, even without considering savings from eliminating any fixed costs of ownership. The reason for this phenomenon is that, under these conditions, the negative effect on the retailer's profit due to double marginalization on the store-brand product is offset by the positive effect of the national brand manufacturer choosing to switch to a strategy of selling a higher volume at a lower per-unit margin than he would have if the retailer retained ownership of the store-brand factory. So the retailer sells less of the store brand at a lower margin and more of the national brand at a higher margin, and the net effect for the retailer is positive.

In addition, we show that the national-brand manufacturer may also benefit from the retailer's decision to outsource; we identify situations in which both the retailer and the national-brand manufacturer benefit. We also identify situations in which the total profit earned by the retailer and the third party manufacturer exceeds the profit that the retailer would earn when producing in-house — and the national-brand manufacturer is better off when the retailer outsources. In such cases, a fixed transfer payment between the retailer and third-party manufacturer can result in all three parties being better off. In the course of our analysis, we also obtain interesting results regarding characteristics of the equilibria under different supply-chain ownership and competitive structures when the store-brand factory has a capacity constraint. To the best of our knowledge, these results have not been presented before in the literature. In particular, our results characterize how wholesale prices and profits for the three parties (the retailer, the national brand, and the store-brand factory owned by a third party) depend on the ownership and competitive position of the store-brand factory and its production capacity.

The next section provides a discussion of the related literature. Section 3 formally presents the model and solves each party's optimization problem. Section 4 characterizes the equilibrium as a function of the production capacity of the store-brand factory. Sections 5 and 6 provide comparative statics and give results on how prices and profits change when the store-brand factory is sold, including a discussion of circumstances in which the sale of the store-brand factory is win-win-win (i.e., all three parties benefit). Section 7 presents a variant of the model in which there is price negotiation between the retailer and third-party manufacturer, and Section 8 contains concluding remarks.

2 Related Literature

We preface this section by noting that, to the best of our knowledge, there is no literature that analyzes the consequence of a retailer selling its capacity-limited store-brand factory. Indeed, there is no literature on the broader topic of how the supply chain structure affects equilibrium (wholesale and retail) prices and the various parties' profits when a retailer offers both store- and national-brands. One reason is that the vast majority of models involving store- and national-brand competition implicitly assume that the store-brand product is produced by a non-strategic third-party that offers the store-brand product at a constant variable cost. In view of the absence of literature directly related to our research, we offer brief overview of the literature on competition between store- and national-brands, and an overview of the literature on vertical disintegration (including outsourcing) in non-store-brand contexts to clarify the broad context into which our research falls.

There is substantial research on competition between store brands and national brands. Due to page limits, we cannot provide a detailed review of the literature and instead refer the reader to a survey paper by Sethuraman (2009) and the book by Kumar and Steenkamp (2007). Our model accounts for the competitive effects and pricing decisions but our focus is on the issue of sourcing: whether the retailer should outsource the production of store-brand products that compete with national brands. Thus, our paper falls into the broader category of vertical integration and disintegration, although we examine a very specific type of disintegration. There is, of course, a long history of research on vertical integration in the economics literature, and on issues related to channel structure in the marketing literature. The topics of these works include vertical integration (Perry, 1989; Joskow, 2005), the make-versus-buy decision (Klein, 2005), investments of buyers in their suppliers (Bensaou and Anderson, 1999), and channel management (Anderson and Coughlan, 2002).

Christensen (1999) discusses factors that favor vertical integration and disintegration and suggests that product modularity and the degree of scale economies are two critical factors. Product modularity does not play a key role in the manufacture of grocery and similar products, but scale economies certainly come into play.

A number of researchers (e.g., Bonanno and Vickers, 1988; Moorthy, 1988), show that under certain conditions involving competing manufacturers, the introduction of a retailer(s) into the supply chain for consumable goods—one form of decentralization—can make a manufacturer better off. (We note, however, that Coughlan and Wernerfelt (1989) argue that these findings rely on the strong assumption that channel pricing agreements are observable to all parties, and that the results may not hold under more realistic assumptions.) Desai et al. (2004) also find that the introduction of a retailer may improve a manufacturer's profit in settings involving durable goods and dynamic decision-making. These results all apply to settings that differ from ours in some fundamental respect, e.g., durable goods, competing retailers, or competing manufacturers.

Liu and Tyagi (2011) show that competing retailers may be better off outsourcing to an independent upstream manufacturer that does not necessarily have a cost advantage. In their model, the product positioning is endogenous, and in equilibrium, greater product differentiation benefits the downstream firms. Chen (2005) shows that a vertically-integrated firm may benefit from spinning off a segment of the firm if the spin-off allows that segment to compete more effectively (due to issues of credibility). We note that Chen's model also involves dynamic decision-making. Durango-Cohen and Wagman (2014) show that an independent capacity-constrained manufacturer may seek to obfuscate its capacity constraint, whereby information about a factory's maximum output may be imperfect; such obfuscation would be less of an issue in our framework because the retailer is informed of its own store-brand factory's capacity.

Extensive literature exists on outsourcing decisions from both the perspective of transaction-cost economics (TCE) and the resource-based view (RBV). For references, see Espino-Rodriguez and Padron-Robaina (2006), Holcomb and Hitt (2007), and Williamson (2008). In these research streams, there is almost always an implicit assumption that the firm making the outsourcing decision is a manufacturer, not a retailer. In our scenario, the choice of the retailer to sell his store-brand factory leads to a different competitive environment than has been envisioned in the economics literature on outsourcing decisions. Kroes and Ghosh (2010) provide a useful overview of the drivers of outsourcing, including agency theory, TCE, RBV and the knowledge-based view. They also perform an empirical study of the congruence of drivers of outsourcing and the firm's competitive priorities, and find that outsourcing congruence is positively correlated with supply-chain performance.

As mentioned earlier, the best of our knowledge, no research has been done on the question of whether a retailer should sell his store-brand factory to a third party. Prior research on competition between national- and store-brands implicitly assumes that the store-brand product is manufactured by an independent party. Phrases such as "producer of the store brand" are common. We also note that when the store-brand and national-brand factories have no capacity limits, the nature of the change in the equilibrium outcome due to the sale of the store-brand factory to a third party is obvious. First, the third-party owner of the store-brand factory will set the wholesale price above cost, leading to double-marginalization where it did not exist before. Second, the national brand, in turn, will raise his wholesale price because of the reduced need to be competitive. Third, the retailer will respond by raising retail prices, thereby driving down demand for both products. For most commonly-used demand functions, the retailer's optimal prices are such that a one dollar change in the wholesale price leads to less than a one dollar change in the retail price. Consequently, this series of changes hurts the retailer because he ultimately sells fewer units of each product and is selling each unit at a lower margin. A key question is thus: How does the limited availability of production capacity affect the retailer's decision regarding whether or not to sell his store-brand factory?

3 Base Models

A retailer sells two competing products: a store brand and a national brand. The store-brand product is produced internally and the factory has limited capacity (our findings extend to a setting where the store-brand factory can source additional capacity at a higher marginal cost). The retailer has the option to sell the factory to a third party. The national brand does not have rigid limits on the capacity that it can dedicate to the retailer. Information is assumed symmetric. As is common in the literature, we assume that the retailer maximizes category profits and does not intentionally set retail prices to protect store-brand market share. A study by Meza and Sudhir (2005) indicates that, under some circumstances, retailers do favor their own brands when setting retail prices. In our context, such a preference could be applied whether or not the retailer owns the store-brand factory. Thus, to isolate the effects of factory ownership and because incorporating favoritism of store brands into a normative model is difficult, we perform our analysis assuming that the retailer sets prices to optimize category profits.

We consider three different game structures in our analysis. In the case of internal production (or IP for short), we assume the national brand is the first mover in a Stackelberg game. When the store brand is produced by a third party, we model competition between the two manufacturers in two ways. The first involves a Nash game between the two suppliers, which we refer to as "third-party (production of the store brand) with Nash (competition between producers)," or $3P^N$ for short, followed by the retailer setting retail prices. Choi (1991) refers to this game as "manufacturer Stackelberg" and Kadilyali et al. (2000) seeks to show it exists empirically. The second way in which we model competition between the two suppliers is the "more traditional" national-brand Stackelberg case, which we refer to as "third-party (production of the store-brand) with Stackelberg leadership (by the national-brand manufacturer)," or $3P^S$ for short. In this scenario, the national-brand manufacturer moves first and the third-party manufacturer moves next in setting wholesale prices, and finally the retailer sets retail prices. The timing of decisions for each of these game structures is shown in Figure 1. We note that the literature provides mixed results on the power relationship between national and store-brand producers so we consider both Nash and Stackelberg competition between the producers. We refer the reader to Narasimham and Wilcox (1998), Steiner (2004) and Berges-Sennou et al. (2004) for surveys on competition between national and store brands.

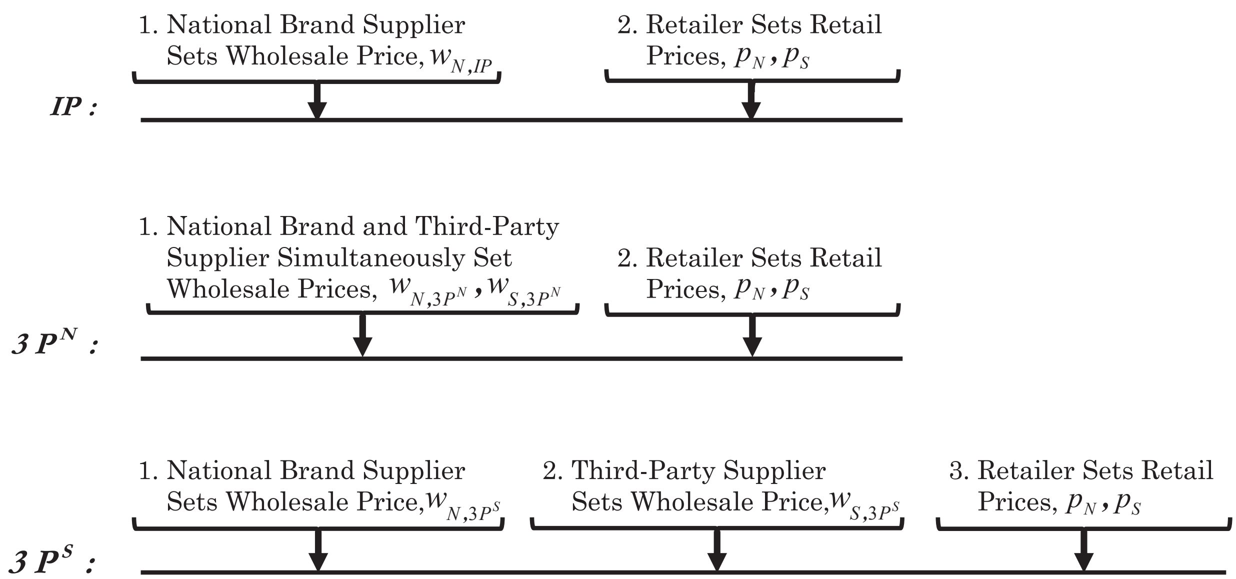

Figure 1: Sequence of Events under IP, $3P^N$ and $3P^S$.

Throughout the paper, for brevity, we use "capacity constrained" or "constrained" to refer to situations where the store-brand factory's capacity constraint is binding. Likewise, we use "capacity unconstrained" or "unconstrained" for situations where the capacity constraint is not binding.

Notation

N, S : index representing national brand and store (or store brand), respectively;

c_S, c_N : unit production (and distribution) cost for store- and national-brand items;

K_S : capacity of the store-brand factory (in units);

Π_R, Π_N, Π_{3P} : retailer, national brand, and third-party producer profits, respectively;

$\alpha_i, \beta_i, \gamma$: parameters of the demand function, $i = N, S$.

Decision variables:

w_N, w_S : wholesale prices offered by the national- and store-brand supplier, respectively;

p_i : retail price for product i, $i \in \{N, S\}$.

We assume retail demands for the two products have the following linear forms:

$$D_S(p_S, p_N) = \alpha_S - (\beta_S + \gamma)p_S + \gamma p_N \quad \text{and} \quad D_N(p_S, p_N) = \alpha_N - (\beta_N + \gamma)p_N + \gamma p_S$$

The parameter γ in the demand functions captures the degree of substitutability of the products. We also make the following technical assumption:

Regularity Condition: For any (c_N, c_S) pair, the store-brand demand is the same for all (p_N, p_S) pairs satisfying $\frac{p_S - c_S}{p_N - c_N} = \frac{\gamma}{(\beta_S + \gamma)}$. This is a very mild regularity condition. If it does not hold, the demand for one or both products is non-positive at the respective production costs, i.e., the product is not profitable to produce, and therefore is not offered.

In the economics literature on differentiated products, and in the economics and marketing literatures involving store-brand products, demand is commonly represented via linear functions (see, e.g., Spence, 1976; Dixit, 1979; Singh and Vives, 1984; Vives, 1985; Sutton, 1998; Vives, 1999). It has also been shown that the specification can be derived from a quadratic representative-consumer welfare function and other microfoundations (Martin, 2009). The underlying rationale is that introducing product variety in terms of substitutes expands overall demand. We cite a few of the many examples of the use of linear demand models from the store-brand literature here. In the classic model of differentiated products of McGuire and Staelin (1983) which readily applies to store brand settings, demands are modeled as linear functions of price. Choi (1991) employs a generalization of this linear demand model. Raju et al. (1995) further generalizes linear demand models to capture special features of national-vs-store brand competition. Cotterill and Putsis (2001) subsequently employs a simpler form of Raju's model, while Sayman et al. (2002) extend Raju's model to accommodate two national brands and one store brand. In view of the longstanding use and justification for linear demand functions, we did not pursue the use of nonlinear demand functions, which, combined with the capacity constraint at the store-brand factory, would have significantly complicated the analysis. However, our approach for deriving the equilibrium is fairly general and relies only on the following mild conditions: (1) the retailer's profit function is jointly concave in the retail prices for the two products; (2) the retailer can drive the demand for either product to zero by setting a sufficiently high price (i.e., the retailer can implicitly choose whether or not to offer the two products via the choice of retail prices); and (3) the retailer responds to changes in the wholesale price(s) in the direction(s) that we would intuitively expect.

We do not require specific relationships between c_N and c_S or between w_N and w_S. Similarly, to maintain generality, we make no assumptions regarding the relationship between p_N and p_S (although in practice, p_N is usually larger than p_S). A study by Davies and Brito (2004) indicates that one main reason for the higher wholesale prices of national brands versus those of third-party suppliers of store brands is the higher costs of the former, partly due to higher advertising expenditures, overhead, and/or labor costs. These higher wholesale prices usually translate into higher retail prices.

3.1 Retailer's Problem

We first consider the retailer's decisions given the wholesale price offered by the national brand, w_N. If the store–brand product is produced internally, the problem is to select p_N and p_S to maximize total profit from both brands, subject to the capacity constraint at the store-brand factory:

$$\max_{p_N, p_S} \Pi_R = (p_N - w_N)\left[\alpha_N - (\beta_N + \gamma)\, p_N + \gamma p_S\right]^+ + (p_S - c_S)\left[\alpha_S - (\beta_S + \gamma)\, p_S + \gamma p_N\right]^+ \quad (1)$$

$$\text{subject to :} \quad \alpha_S - (\beta_S + \gamma)\, p_S + \gamma p_N \leq K_S$$

The retailer's unconstrained optimal price reaction functions are (see Appendix A for the standard first-order conditions):

$$p_N^* = \frac{\alpha_N(\beta_S + \gamma) + \gamma\alpha_S}{2\,(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)} + 0.5 w_N \quad \text{and} \quad p_S^* = \frac{\alpha_S(\beta_N + \gamma) + \gamma\alpha_N}{2\,(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)} + 0.5 c_S \quad (2)$$

Notice that the above expressions contain w_N, which is decided by the national-brand manufacturer. Thus, until the national-brand manufacturer's wholesale price decision (which we study in the next section) is known, we cannot know whether the above prices will keep the store-brand demand below the store-brand factory's capacity. If the retailer's unconstrained optimal response to the national-brand manufacturer's unconstrained optimal wholesale price would cause the store-brand capacity to be exceeded, it is optimal for the retailer to set the retail prices to ensure that the capacity constraint is satisfied as an equality, leading to the following retail prices:

$$p_N^* = \frac{\alpha_N(\beta_S + \gamma) + \alpha_S}{2\,(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)} + 0.5 w_N \quad (3)$$

$$p_S^* = \frac{\alpha_S(\beta_N + \gamma) + \gamma\alpha_N}{2(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)} + \frac{\gamma}{2(\beta_S + \gamma)} w_N + \frac{\alpha_S - 2K_S}{2(\beta_S + \gamma)} \quad (4)$$

The capacity constraint at the store-brand factory needs to be handled similarly for the other game structures as well.

If the third party produces the store-brand product, the retailer's problem is:

$$\max_{p_N, p_S} \Pi_R = (p_N - w_N)\left[\alpha_N - (\beta_N + \gamma)\, p_N + \gamma p_S\right]^+ + (p_S - w_S)\left[\alpha_S - (\beta_S + \gamma)\, p_S + \gamma p_N\right]^+ \quad (5)$$

We note that in this case, the retailer need not worry about the constraint on factory capacity. Instead, the third-party producer sets w_S, ensuring that the capacity constraint is satisfied (else the third party sets a suboptimally low wholesale price and fails to maximize profit). The optimal retail prices in this case are:

$$p_N^* = \frac{\alpha_N(\beta_S + \gamma) + \gamma\alpha_S}{2\,(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)} + 0.5 w_N \quad \text{and} \quad p_S^* = \frac{\alpha_S(\beta_N + \gamma) + \gamma\alpha_N}{2\,(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)} + 0.5 w_S \quad (6)$$

3.2 National Brand's Problem

The national brand's pricing problem depends upon whether the store brand product is produced internally (internal production, or *IP* for short), which we consider first, or by a third party. We denote by $3P^N$ the case of Nash competition between the two external suppliers; we denote by $3P^S$ the Stackelberg equilibrium, in which the national brand chooses its wholesale price first, followed by the third-party producer.

3.2.1 Internal Production of Store Brand (IP)

The national brand faces the following profit-maximization problem:

$$\max_{w_N} \Pi_N^{IP} = (w_N - c_N)\left[\alpha_N - (\beta_N + \gamma)\, p_N + \gamma p_S\right]$$

where p_N^* and p_S^* are defined by (2) and (3), respectively. This represents the first stage in the game: the national-brand manufacturer chooses a wholesale price, w_N, recognizing that the retailer will next choose retail prices to optimize his total profit from both products, as described in Section 3.1. The national brand's problem is well-structured, although his optimal price depends upon whether he can profitably utilize his first-mover advantage to "force" the store-brand factory to operate at capacity (which usually yields less profit for the retailer). The national brand may be better off taking a less aggressive stance and not forcing the store-brand factory to operate at capacity. The detailed analysis appears in the unabridged version of this paper; we present the essential results in Table 1.

Table 1: Equilibrium Wholesale Prices, Demands and Profits under IP

Capacity Configuration	Store-Brand Equilibrium
Unconstrained	$w_{N,IP}^{U*} = \dfrac{\alpha_N + \gamma c_S}{2(\beta_N + \gamma)} + 0.5 c_N$ $D_{N,IP}^{U*} = \dfrac{\alpha_N - (\beta_N + \gamma) c_N + \gamma c_S}{4}$ $D_{S,IP}^{U*} = \dfrac{\gamma(\alpha_N + (\beta_N + \gamma) c_N + \gamma c_S) + 2(\beta_N + \gamma)(\alpha_S - (\beta_S + \gamma) c_S)}{4(\beta_N + \gamma)}$ $\Pi_{N,IP}^{U*} = \dfrac{(\alpha_N - (\beta_N + \gamma) c_N + \gamma c_S)^2}{8(\beta_N + \gamma)}$ $\Pi_{R,IP}^{U*} = \dfrac{1}{16(\beta_N + \gamma)} \Bigg[\alpha_N^2 + \dfrac{4\left[\gamma \alpha_N + (\beta_N + \gamma)\alpha_S\right]^2}{(\beta_N \beta_S + \beta_N \gamma + \beta_S \gamma)}$ $+ \left[(\beta_N + \gamma)^2 c_N - 2(\beta_N + \gamma)(\alpha_N + \gamma c_S)\right] c_N$ $+ \left(\left(4(\beta_N \beta_S + \beta_N \gamma + \beta_S \gamma) + \gamma^2\right) c_S - 2\left(4(\beta_N + \gamma)\alpha_S + 3\gamma \alpha_N\right)\right) c_S \Bigg]$
Constrained	$w_{N,IP}^{C*} = \dfrac{\gamma(\alpha_S - 2K_S) + (\beta_S + \gamma)\alpha_N}{2(\beta_N \beta_S + \beta_N \gamma + \beta_S \gamma)} + 0.5 c_N$ $D_{N,IP}^{C*} = \dfrac{(\beta_S + \gamma)\alpha_N - (\beta_N \beta_S + \beta_N \gamma + \beta_S \gamma) c_N + \gamma(\alpha_S - 2K_S)}{4(\beta_S + \gamma)}$ $D_{S,IP}^{C*} = K_S$ $\Pi_{N,IP}^{C*} = \dfrac{\left[(\beta_S + \gamma)\alpha_N - (\beta_N \beta_S + \beta_N \gamma + \beta_S \gamma) c_N + \gamma(\alpha_S - 2K_S)\right]^2}{8(\beta_S + \gamma)(\beta_N \beta_S + \beta_N \gamma + \beta_S \gamma)}$ $\Pi_{R,IP}^{C*} = \dfrac{1}{4(\beta_S + \gamma)(\beta_N \beta_S + \beta_N \gamma + \beta_S \gamma)} \Big(\left[4(\beta_N \beta_S + \beta_N \gamma + \beta_S \gamma) + 3\gamma^2\right](\alpha_S - K_S)$ $+ 3\gamma(\beta_S + \gamma)\alpha_N - 4(\beta_S + \gamma)(\beta_N \beta_S + \beta_N \gamma + \beta_S \gamma) c_S + \gamma(\beta_N \beta_S + \beta_N \gamma + \beta_S \gamma) c_N \Big) K_S$ $+ \dfrac{\left[(\gamma \alpha_S + (\beta_S + \gamma)\alpha_N) - (\beta_N \beta_S + \beta_N \gamma + \beta_S\ \gamma) c_N\right]^2}{16(\beta_S + \gamma)(\beta_N \beta_S + \beta_N \gamma + \beta_S \gamma)}$

In the remainder of the paper, let $w_{N,x}^{l*}$, $D_{N,x}^{l*}$ and $\Pi_{N,x}^{l*}$, where $x \in \{IP, 3P^N, 3P^S\}$, $l \in \{U, C\}$ denote the equilibrium wholesale price, demand, and profit for the national-brand supplier under the different supply-chain configurations (or "configurations" for short), where the superscripts U

and C refer to the unconstrained and constrained equilibrium settings, respectively. The demand for the store-brand product is denoted by $D^{l*}_{S,x}$. Similarly, $w^{l*}_{S,3P^N}$ and $w^{l*}_{S,3P^S}$ are the equilibrium wholesale prices for the store brand offered by the third-party supplier under the $3P^N$ and $3P^S$ settings, respectively, for $l \in \{U, C\}$. The equilibrium profits for the third-party supplier and retailer, respectively, are denoted by $\Pi^{l*}_{S,x}$ and $\Pi^{l*}_{R,x}$, $x \in \{IP, 3P^N, 3P^S\}$, $l \in \{U, C\}$.

Using the results in Table 1, it can be shown that if $\Pi^{U*}_{N,IP} \geq \Pi^{C*}_{N,IP}$, the national brand prefers that the store-brand factory operates below capacity, and will choose w_N to induce him to do so. Substituting the profit expressions above and simplifying, the inequality is given by:

$$\begin{aligned} K_S \;\; &\geq \;\; \frac{1}{2\gamma}\Bigg\{(\beta_S+\gamma)\alpha_N - (\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)c_N + \gamma\alpha_S \\ &\quad - \sqrt{\frac{(\beta_S+\gamma)(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)(\alpha_N-(\beta_N+\gamma)c_N+\gamma c_S)^2}{(\beta_N+\gamma)}}\Bigg\} \equiv K_S^{IP}. \end{aligned} \tag{7}$$

3.2.2 Third-Party Production — Nash Competition ($3P^N$)

When the store-brand product is produced by a third party, that firm must compete with the national brand. The (Nash) equilibrium wholesale prices must reflect the capacity constraint at the store-brand factory and the retailer's choice of prices as a function of both wholesale prices.

For a given w_j, $j \in \{S, N\}$, and anticipating the retailer's response to both wholesale prices, supplier $i \neq j$ would solve:

$$\max_{w_i} \Pi_{i,3P^N} \;=\; (w_i - c_i)\left[\alpha_i - (\beta_i+\gamma)\left(\tfrac{\alpha_i(\beta_j+\gamma)+\gamma\alpha_j}{2(\beta_i\beta_j+\beta_i\gamma+\beta_j\gamma)} + 0.5w_i\right) + \gamma\left(\tfrac{\alpha_j(\beta_i+\gamma)+\gamma\alpha_i}{2(\beta_i\beta_j+\beta_i\gamma+\beta_j\gamma)} + 0.5w_j\right)\right]^+ \tag{8}$$

The third party's capacity constraint is given by:

$$\alpha_S - (\beta_N+\gamma)\left(\frac{\alpha_S(\beta_N+\gamma)+\gamma\alpha_N}{2(\beta_S\beta_N+\beta_S\gamma+\beta_N\gamma)} + 0.5w_S\right) + \gamma\left(\frac{\alpha_N(\beta_S+\gamma)+\gamma\alpha_S}{2(\beta_S\beta_N+\beta_S\gamma+\beta_N\gamma)} + 0.5w_N\right) \leq K_S$$

In the above, we simplified the expressions by substituting in the retailer's optimal prices. This allows us to highlight the dependence of demand on suppliers' chosen wholesale prices. As in the earlier case, the equilibrium prices depend upon the capacity limitation of the third-party supplier in equilibrium. Detailed results appear in Appendix B; key results appear in Table 2 below.

The value of K_S that equates $w^{U*}_{N,3P^N}$ and $w^{C*}_{N,3P^N}$ (in Table 2) is the threshold value of K_S at which the store-brand factory becomes constrained. It follows that the third-party supplier's capacity constraint is not binding in equilibrium if

$$K_S \geq \frac{\frac{1}{2}(\beta_S+\gamma)\left[2(\beta_N+\gamma)\alpha_S - (2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)c_S + (\beta_N+\gamma)\gamma c_N + \gamma\alpha_N\right]}{4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2} \equiv K_S^{3P^N} \tag{9}$$

Table 2: Equilibrium Wholesale Prices and Demands under *3P*^{N}

Store-Brand Equilibrium	Capacity Configuration	Unconst.	$w^{U^*}_{N,3P^N} = \dfrac{\gamma\alpha_S + (\beta_S+\gamma)[2\alpha_N + \gamma c_S + 2(\beta_N+\gamma)c_N]}{4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2}$
			$w^{U^*}_{S,3P^N} = \dfrac{\gamma\alpha_N + (\beta_N+\gamma)[2\alpha_S + \gamma c_N + 2(\beta_S+\gamma)c_S]}{4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2}$
			$D^{U^*}_{N,3P^N} = \dfrac{(\beta_N+\gamma)\left[2(\beta_S+\gamma)\alpha_N + \gamma(\alpha_S + (\beta_S+\gamma)c_S) - (2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)c_N\right]}{2[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]}$
			$D^{U^*}_{S,3P^N} = \dfrac{(\beta_S+\gamma)\left[2(\beta_N+\gamma)\alpha_S + \gamma(\alpha_N + (\beta_N+\gamma)c_N) - (2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)c_S\right]}{2[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]}$
		Constr.	$w^{C^*}_{N,3P^N} = \dfrac{\gamma(\alpha_S - 2K_S) + (\beta_S+\gamma)[\alpha_N + (\beta_N+\gamma)c_N]}{2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2}$
			$w^{C^*}_{S,3P^N} = \dfrac{\gamma\alpha_N + (\beta_N+\gamma)(2\alpha_S + c_N\gamma - 4K_S)}{2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2}$
			$D^{C^*}_{N,3P^N} = \dfrac{(\beta_N+\gamma)\left[(\beta_S+\gamma)\alpha_N - (\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)c_N + \gamma(\alpha_S - 2K_S)\right]}{2[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]}$
			$D^{C^*}_{S,3P^N} = K_S$

3.2.3 Third-Party Production — Stackelberg Equilibrium (*3P*^{S})

We first study the third-party supplier's response to any w_N offered by the national brand. The third party supplier seeks to maximize:

$$\begin{aligned}\max_{w_S} \Pi_{S,3P^S} &= (w_S - c_S)\left\{\alpha_S - (\beta_S+\gamma)\left[\frac{\alpha_S(\beta_N+\gamma)+\gamma\alpha_N}{2(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} + 0.5w_S\right] + \right.\\ &\qquad \left. \gamma\left[\frac{\alpha_N(\beta_S+\gamma)+\gamma\alpha_S}{2(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} + 0.5w_N\right]\right\}\\ \text{subject to:} &\quad \alpha_S - (\beta_S+\gamma)p_S + \gamma p_N \le K_S\end{aligned}$$

Thus, $w_S^* = \dfrac{\alpha_S}{2(\beta_S+\gamma)} + \dfrac{\gamma w_N}{2(\beta_S+\gamma)} + 0.5c_S$. The national brand's problem then becomes:

$$\begin{aligned}\max_{w_N} \Pi_{N,3P^S} &= (w_N - c_N)\left\{\alpha_N - (\beta_N+\gamma)\left[\frac{\alpha_N(\beta_S+\gamma)+\gamma\alpha_S}{2(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} + 0.5w_N\right]\right.\\ &\qquad \left. + \gamma\left[\frac{\alpha_S(\beta_N+\gamma)+\gamma\alpha_N}{2(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} + \frac{1}{2}\left(\frac{\alpha_S}{2(\beta_S+\gamma)} + \frac{\gamma w_N}{2(\beta_S+\gamma)} + 0.5c_S\right)\right]\right\}\end{aligned}$$

Solving the national brand's problem, we obtain the equilibrium wholesale prices as shown in Table 3. Detailed derivations appear in Appendix C. Because the national brand is the first mover, he can choose the equilibrium of the continuation subgame that he prefers. The national brand prefers the equilibrium in which the store-brand factory is unconstrained if $\Pi^{U^*}_{N,3P^S} \ge \Pi^{C^*}_{N,3P^S}$.

Thus far, we have made some preliminary observations that provide the foundation for the proceeding analysis. We next compare the equilibrium capacity configurations (Section 4), and identify conditions under which the retailer is better off by selling the factory (Section 5).

Table 3: Equilibrium Wholesale Prices and Demands under *3P*^{S}

Store-Brand Equilibrium	Capacity Configuration	
	Unconstrained	$w^{U^*}_{N,3P^S} = \frac{c_N}{2} + \frac{\gamma\alpha_S + (\beta_S+\gamma)\left[2\alpha_N + \gamma c_S\right]}{4(\beta_N+\gamma)(\beta_S+\gamma) - 2\gamma^2}$
		$w^{U^*}_{S,3P^S} = \frac{\gamma c_N}{4(\beta_S+\gamma)} + \frac{(4(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)\left[\alpha_S + (\beta_S+\gamma)c_S\right] + 2(\beta_S+\gamma)\gamma\alpha_N}{4[(2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)(\beta_S+\gamma)]}$
		$D^{U^*}_{N,3P^S} = \frac{2(\beta_S+\gamma)\alpha_N + \gamma(\alpha_S + (\beta_S+\gamma)c_S) - (2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)c_N}{8(\beta_S+\gamma)}$
		$D^{U^*}_{S,3P^S} = \frac{2(\beta_S+\gamma)\left[\gamma\alpha_N - (4(\beta_N+\gamma)(\beta_S+\gamma) - 3\gamma^2)c_S\right] + (4(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)\alpha_S}{8(2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)} + \frac{(2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)c_N}{8(2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)}$
	Constrained	$w^{C^*}_{N,3P^S} = w^{C^*}_{N,IP}$
		$w^{C^*}_{S,3P^S} = \frac{\gamma c_N}{2(\beta_S+\gamma)} + \frac{[2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2](\alpha_S - 2K_S) + \gamma(\beta_S+\gamma)\alpha_N}{2(\beta_S+\gamma)(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)}$
		$D^{C^*}_{N,3P^S} = D^{C^*}_{N,IP}$
		$D^{C^*}_{S,3P^S} = K_S$

4 Comparison of Equilibrium Capacity Configurations

The capacity thresholds derived in the previous section give rise to regions in which the store-brand factory is (or is not) capacity constrained under *IP*, *3P*^{N} and *3P*^{S} in Figure 2. We first establish the relationship among these capacity thresholds, and then use these results to study how the wholesale price offered by the national brand varies under the different configurations.

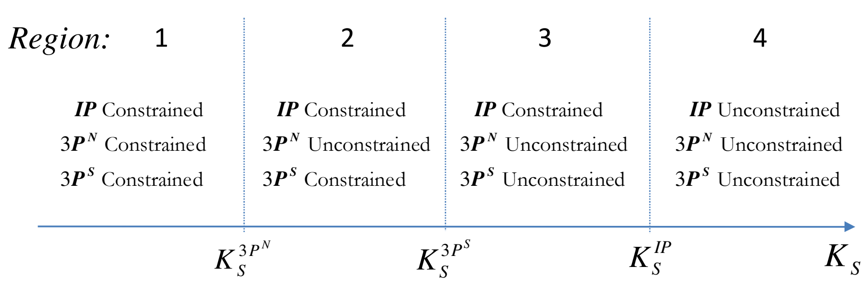

Figure 2: Equilibrium configurations under IP, *3P*^{N}, and *3P*^{S} as a function of K_S.

In Figure 2, the capacity $K_S^{3P^N}$ denotes the level below which the store-brand factory is capacity constrained under third-party ownership with Nash competition, while $K_S^{3P^S}$ and K_S^{IP} are the analogous capacity levels under Stackelberg competition and under internal production, respectively. In Appendix D, we show that $K_S^{3P^N} \leq K_S^{3P^S} \leq K_S^{IP}$. The intervals delimited by these thresholds define four regions in the K_S-space. Within each region, we know whether the store-brand factory's capacity constraint is binding depending on the configuration.

We next compare the national brand's wholesale prices under *IP*, *3P*^{N} and *3P*^{S}, and show how prices are affected by the capacity of the store-brand factory.

4.1 Comparison of the National Brand's Equilibrium Wholesale Prices

Clearly, the introduction of an independent third-party producer will lead to double marginalization on the store-brand product. However, it is not immediately clear how this will affect the national brand's behavior, especially when capacity constraints come into play.

The next set of results shows how the national brand's wholesale price changes when the store-brand factory is capacity-constrained. In the interest of brevity, the results are illustrated in Figure 3; the proofs are provided in Appendix E. Unsurprisingly, the national brand's wholesale price is smaller under $3P^N$ than under $3P^S$ due to the loss of his first-mover advantage in the former case, and this is true for all capacity levels of the store-brand factory.

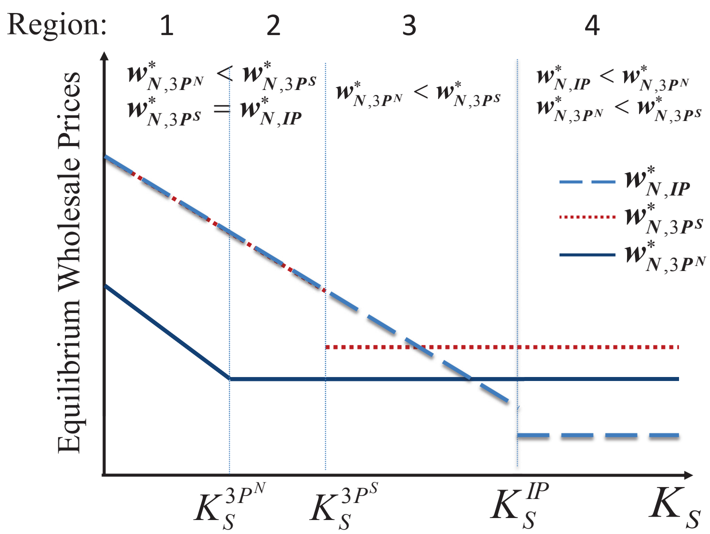

Figure 3: Equilibrium prices under IP, $3P^N$, and $3P^S$ configurations as a function of K_S.

For low capacity levels ($K_S \leq K_S^{3P^S}$), the national brand's equilibrium wholesale price is the same under $3P^S$ and IP because the national brand can utilize his Stackelberg leadership to his advantage. At moderate capacity levels ($K_S^{3P^S} \leq K_S \leq K_S^{IP}$), the national brand's equilibrium price under IP may be larger than or smaller than his Stackelberg and Nash equilibrium prices for the third-party ownership scenarios. Furthermore, as can be observed from the figure, there exists a threshold capacity level below (above) which the national brand's wholesale price is smaller (larger) under third-party ownership than under retailer ownership of the store-brand factory.

If the store-brand factory has ample capacity ($K_S \geq K_S^{IP}$), the national brand's equilibrium wholesale price *increases* under third-party production with either Nash or Stackelberg competition.

Taken together, these findings indicate that the capacity level plays a complex role in influencing whether, in what direction, and how much the national brand changes his wholesale price when the store-brand factory is sold to a third party. Indeed, it is surprising that double marginalization of the store-brand product may lead to a *decline* in the wholesale price of the national-brand product.

We next examine how the capacity of the store-brand factory affects the retailer's profit and whether the retailer may be better off if he sells the store-brand factory.

5 Capacity and the Retailer's Profit

The change in the retailer's profit due to selling the store-brand factory depends heavily on the factory's capacity. In this section, we explore how the retailer's profit differs between the *IP* and $3P^N$ configurations as a function of the store brand factory's capacity level (Results for the $3P^S$ configuration appear in Appendix C). One of our goals is to identify conditions under which the retailer is better off or only marginally worse off under $3P^N$ than under *IP*. We recall that our model does not account for any fixed costs of ownership; thus, if the retailer is only marginally worse off in our model, he may be better off when all costs are considered.

Before proceeding with the analysis, we first present some properties of the retailer's profit functions. The retailer's profits under IP are listed in Table 1. The retailer's profits under the $3P^N$ and $3P^S$ configurations can be similarly derived using the prices in (2)-(4) and the expressions for wholesale prices and demands in Tables 2 and 3. The proof of the following result is in Appendix F.1.

Lemma 1 *Over the regions in Figure 3, the retailer's profit functions have the following forms:*
(a) Under $3P^N$*: convex increasing in Region 1 and constant in Regions 2, 3, and 4.*
(b) Under IP: concave increasing in Regions 1, 2 and 3; an (upward) jump-discontinuity at the upper boundary of Region 3; constant in Region 4.
(c) Under $3P^S$*: convex increasing in Region 1 and 2; an (upward) jump-discontinuity at the upper boundary of Region 2; and constant in Regions 3 and 4.*

We next compare the retailer's profit under the different configurations.

5.1 Comparison of Retailer's Profit in Regions 1 and 2

Recall that $w^{C*}_{N,3P^N}$ and $w^{U*}_{N,3P^N}$ in Regions 1 and 2, respectively, are smaller than $w^{C*}_{N,IP}$, as proved earlier. Therefore, if the national brand's price under $3P^N$ is sufficiently smaller than it is under *IP*, the retailer's profit from the national-brand product may sufficiently rise to offset the effect of double marginalization on the store-brand product. We formalize this possibility below.

Proposition 1 *There exists an interval of capacity levels in Region 1 over which the retailer's profit is always higher under* $3P^N$ *than under IP.*

The proof is in Appendix F.2; a brief sketch of the proof follows. First, we show that the retailer's profit under *3P*^{N} exceeds his profits under *IP* at $K_S = 0$. From Lemma 1, it then follows that the retailer's profit functions under the *IP* and *3P*^{N} configurations may cross at most twice, as one is convex increasing and the other is concave increasing. Together, these results imply that the retailer will always be better off under *3P*^{N} than under *IP* for some K_S-interval in Region 1. Figure 4 illustrates how the retailer's profit may compare across the two configurations. It also shows how the retailer's profits may compare under *IP* and *3P*^{N} in Region 2.

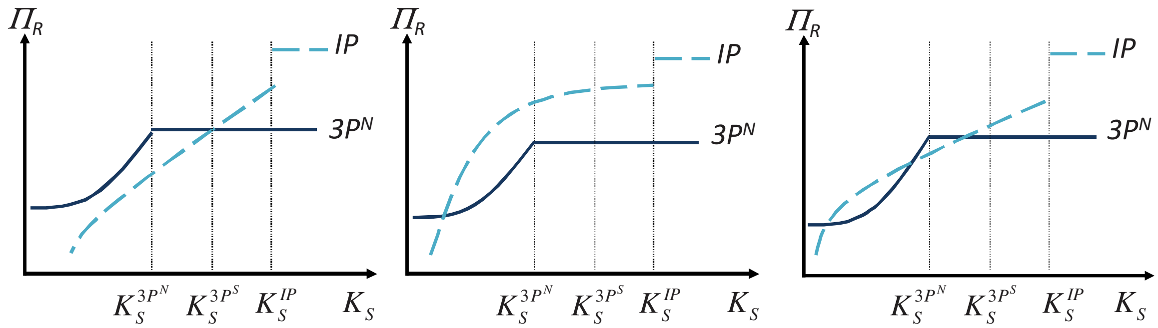

Figure 4: Potential crossings of the retailer's profit functions under *IP* and *3P*^{N}.

Corollary 1: *The retailer's profit under 3P*^{N} *may exceed his profit under IP for capacity levels in Region 2.*

Under the *3P*^{S} scenario, the national brand, due to his first-mover advantage, is able to command the same wholesale price as in the *IP* configuration, and obtains the same level of profit. By substituting for the wholesale prices in (3)-(5), we find that the retailer charges the same retail prices under *3P*^{S} as under *IP* for both products (and, therefore, faces the same demand as in the IP configuration); however, the retailer's profit always decreases due to double marginalization on the store-brand product, as stated formally in the next proposition.

Proposition 2 *The retailer's profit under 3P*^{S} *never exceeds his profit under IP for capacity levels in Regions 1 and 2.*

5.2 Comparison of Retailer's Profit in Region 3

The above results demonstrate that the retailer's profit function under IP dominates the retailer's profit function under *3P*^{S} over the first two regions. At the lower boundary of Region 3, where the equilibrium capacity configuration changes from constrained to unconstrained, there is a discontinuity (an upward jump) in the retailer's profit function. Due to this discontinuity, the retailer may be better off under the *3P*^{S} configuration in all or a portion of Region 3, as the following proposition and Figure 5 indicate.

Proposition 3 *The retailer may be better off (profit-wise) under $3P^S$ than under IP for some capacity levels in Region 3.*

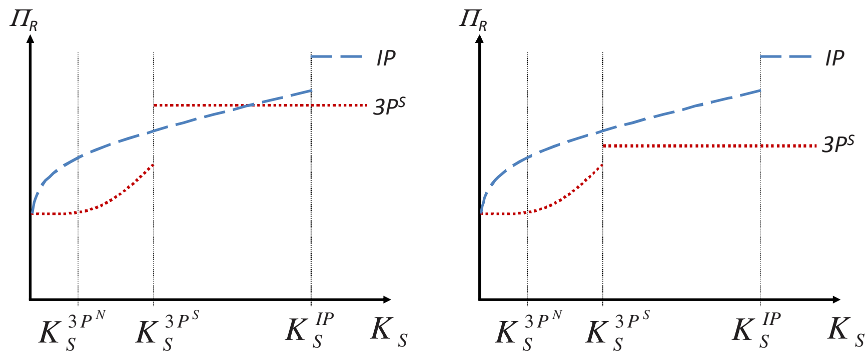

Figure 5: Potential crossings of retailer's profit functions under IP and $3P^S$.

The retailer may also be better off under $3P^N$ than under IP in Region 3. This follows from the fact that in Region 3, the retailer's profit under IP is increasing, whereas it is constant under $3P^N$. Thus, they may cross at most once.

5.3 Comparison of Retailer's Profit in Region 4

In Region 4, the equilibrium capacity configuration is unconstrained under all three settings, and w_N^* is greater under third-party production than under internal production. As a result, the retailer will be worse off if he sells the factory. This result is intuitive: a dollar increase in the wholesale price leads to less than a dollar increase in the optimal retail price. Due to double marginalization on the store-brand product and the wholesale-price increase of the national-brand product, the retailer will sell smaller quantities of both products at smaller margins, strictly reducing his profit.

5.4 Implications of the Findings for the Four Regions

One inference that can be made from the foregoing results in this section is that conditions need to be somewhat extreme in terms of the (weak) quality of the store-brand product or in terms of the capacity tightness of the store-brand factory, or both, to make it economically advantageous for the retailer to sell the store-brand factory. Nevertheless, it is surprising that the introduction of a capacity constraint at the retailer's factory, which always exists in practice, can lead to a conclusion that would **never** occur in the absence of a capacity constraint. Furthermore, as noted in the introduction, some retailers own store-brand factories that are operating near capacity, and these retailers offer some store-brand products that are intentionally positioned to be at a much lower quality tier than the national brand and offered at a much lower price. One example of such a product category is store-brand soft drinks. Thus, real situations exist that fit the conditions under which the retailer would benefit from selling the store-brand factory.

Our results imply that for store-brand products of sufficiently high quality that are produced in a facility without tight capacity constraints, the retailer is better off keeping (or bringing) production in-house (if the retailer can cover the fixed cost of operation with the incremental profit from production in-house). Some major retailers, such as Kroger and HEB, which have sufficient capital to invest in manufacturing facilities, have identified product lines that fit these specifications and are actively building or purchasing production facilities (cf. CBS News (2009) and Nowlin (2014)).

5.5 Effect of Degree of Substitutability

The results described above hold whether the products are close substitutes or not. In other words, although the retailer achieves a higher profit as product substitutability (γ) increases, a low substitutability does not preclude the retailer from benefiting from the sale of his store-brand factory. This is because even for low values of γ, the positive effect of competition may dominate the negative effect of double marginalization. We provide an example to illustrate this result in Appendix F.4.

We next explore how the national brand's profit and market share are affected by the supply-chain configuration and the capacity of the store-brand factory.

6 Implications for the Manufacturers

6.1 The National Brand

In this subsection, we characterize the national brand's profits and equilibrium demand under each configuration, and then compare them across the different store-brand capacity regions. We begin with the following technical result (proofs for this section are in Appendix H).

Lemma 2 *Under IP, the national brand's profit is convex decreasing in K_S in Regions 1, 2 and 3, and constant in Region 4. Under $3P^S$, the national brand's profit is convex decreasing in K_S in Regions 1 and 2, and constant in Regions 3 and 4. Under $3P^N$, the national brand's profit function is convex decreasing in K_S in Region 1 and constant in Regions 2, 3, and 4. Furthermore, each profit function is continuous.*

The next result shows how the national brand's profits compare under the different configurations.

Lemma 3 *In Regions 1, $\Pi^{C*}_{N,IP} = \Pi^{C*}_{N,3P^S} > \Pi^{C*}_{N,3P^N}$. In Region 2, $\Pi^{C*}_{N,IP} = \Pi^{C*}_{N,3P^S} > \Pi^{U*}_{N,3P^N}$. In Region 3, $\Pi^{U*}_{N,3P^S} > \Pi^{U*}_{N,3P^N}$ and $\Pi^{U*}_{N,3P^S} > \Pi^{C*}_{N,IP}$. In Region 4, $\Pi^{U*}_{N,3P^S} \geq \Pi^{U*}_{N,3P^N} \geq \Pi^{U*}_{N,IP}$.*

Figure 6(a) applies Lemmas 2 and 3 in depicting the national brand's profit under the different configurations. The relationship shown in Region 4 ($K_S \geq K_S^{IP}$) is what we would intuitively expect in the absence of capacity constraints: it parallels the price-setting power of the national brand. But some of the relationships in the other regions are not necessarily intuitive, and these phenomena

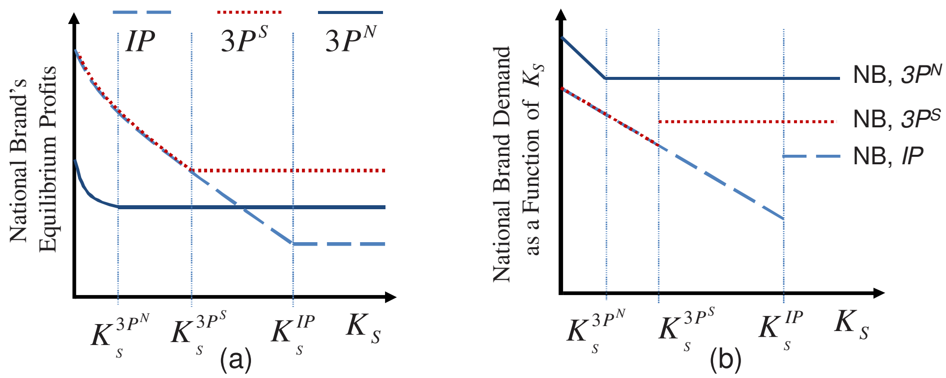

Figure 6: (a) National brand's equilibrium profits as a function of K_S; (b) National brand's equilibrium demand as a function of K_S.

can also be attributed to the national brand's pricing power. We note the similarity between Figure 2, which shows the national brand's equilibrium prices, and Figure 5(a), which shows the national brand's profits. The only qualitative difference is that some portions of the profit functions have curvatures that arise from the nonlinear effect of multiplying price and volume.

These findings show that the national brand's profit depends not only on whether he is the first mover, but also on the ownership and capacity of the store-brand factory. Indeed, for any supply-chain configuration, increasing the capacity of the store-brand factory puts pressure on the national brand to reduce his price, up to the point where the capacity constraint is no longer binding. Since the threshold at which the store-brand factory is no longer capacity constrained varies across the supply-chain configurations, some unexpected relationships arise.

Interestingly, from Figures 5(a) and 6(a), we observe that at some capacity levels within Region 3, both the retailer and the national brand manufacturer may be better off if the retailer sells the store-brand factory. We elaborate further on this in Section 6.3.

We now turn our attention to the impact of selling the factory on the national brand's equilibrium demand (depicted in Figure 6(b)).

Lemma 4 *In Regions 1,* $D^{C*}_{N,IP} = D^{C*}_{N,3P^S} < D^{C*}_{N,3P^N}$. *In Region 2,* $D^{C*}_{N,IP} = D^{C*}_{N,3P^S} < D^{U*}_{N,3P^N}$. *In Regions 3,* $D^{C*}_{N,IP} \leq D^{U*}_{N,3P^S} \leq D^{U*}_{N,3P^N}$. *In Region 4,* $D^{U*}_{N,IP} \leq D^{U*}_{N,3P^S} \leq D^{U*}_{N,3P^N}$.

From Figure 6(b), we can see that if the national brand's primary concern is unit sales (demand), he would prefer that the retailer sells his factory (and to compete Nash with the third party). This finding is irrespective of the capacity level of the store-brand factory. We note that this is the opposite of his preference ordering when considering profits. Said another way, the national brand can trade off high profit for high demand, and vice versa.

6.2 Third-Party Manufacturer

We show (details are in Appendix I) that the third–party manufacturer prefers the national brand to be the Stackelberg leader when the store-brand capacity is in Regions 1, 3 or 4. We have not been able to establish the result for Region 2 due to the complex forms of the expressions for the region boundaries, but have observed from numerical examples that Region 2 is quite small, and have been unable to generate a counterexample in which the third–party prefers to be the leader. This relationship is illustrated in Figure 7. In view of these results, both parties may be better off with such an arrangement. The main reason for this phenomenon is that the national brand charges a higher price when he is the Stackelberg leader than under Nash competition, which gives the third party manufacturer more latitude in maintaining a relatively high wholesale price.

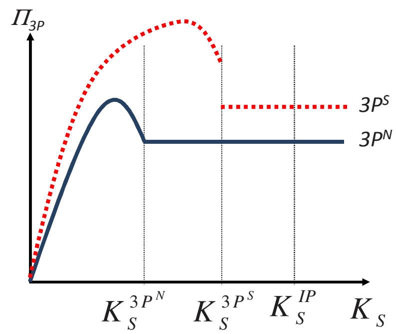

Figure 7: Third Party's Profits under $3P^N$ and $3P^S$ configurations.

6.3 Win-Win-Win Scenarios

We have been able to identify instances in which all parties (the retailer, the national brand, and the third party) are better off as a result of the retailer selling its store-brand factory. From Figure 6(a), it is evident that there cases, particularly in Region 3, where the national brand prefers to operate under the $3P^S$ configuration. The retailer, as demonstrated in Figure 5(a), may also prefer the $3P^S$ configuration to the IP configuration in Region 3. These situations are driven by a switch from a constrained equilibrium under IP to an unconstrained equilibrium under $3P^S$. Relative to the constrained equilibrium under IP, where the store brand is sold more aggressively (with the retailer only paying the marginal cost rather than the wholesale price set by the third party), this switch enables the national brand to adopt a strategy of expanding market share by offering a lower wholesale price. As a result, the national brand captures a larger market share and is better off. The retailer, for his part, is selling overall larger quantities at slightly lower retail prices, and is overall better off. Finally, the third party is trivially better off since its profit is non-zero. Because of the complex parameter interactions in the retailer and national-brand profit functions, it is rather difficult to identify exact parameter ranges for such win-win-win scenarios. From our exploration of numerical examples, we have found that these cases arise when demand for the two products is strong (i.e., the α values are large) but very price-sensitive, the two products are highly substitutable, and the store-brand has a cost disadvantage relative to the national brand.

In such circumstances, if the retailer produces in-house, he sets a high retail price for the store brand — not only to cover his production cost but also to keep demand for the store brand below capacity. Due to high substitutability with the store-brand product and anticipating the retailer's actions, the national brand finds it optimal to set a relatively high wholesale price.

If, on the other hand, the store brand is produced by a third party, the double marginalization on top of the high production cost causes the retailer to favor selling less of the store brand and more of the national brand. Recognizing this, the national brand manufacturer lowers his wholesale price, and is able to garner significantly higher market share because demand is highly price-sensitive.

The combination of conditions described above would be applicable in the case of a small retail chain whose factory cannot reach sufficient economies of scale to achieve low production costs, producing a high-volume commodity product. Examples include granulated sugar and flour for baking. Products of this type are sold in relatively high volumes. Moreover, they are essentially commodities, so the national- and store-brands are highly substitutable and demand is highly price-sensitive. Indeed, even retail chains of moderate size cannot achieve sufficient scale economies in the production of this type of product, so they almost uniformly choose to outsource them. (Neither Safeway nor Kroger owns a factory to mill grains but both sell private label flour. See Innate Health Services, LLC (2010) regarding sugar processors and their private label relationships.) As products in some categories become increasingly commoditized, such situations are likely to become more common, even for products for which national and store brands are perceived as differentiated at this time.

7 Price Negotiations with the Third-Party Manufacturer

In this section, we study a situation in which the third-party store-brand manufacturer has incomplete power in setting his wholesale price (e.g., due to preexisting contractual terms with the retailer). We do so by embedding a negotiation subgame between the retailer and the store-brand manufacturer to determine the resulting store-brand wholesale price. We model this subgame as a random-proposer game (cf. Marx and Shaffer 2007, 2010; Krasteva and Yildirim 2012). We assign a bargaining power φ to the retailer and $1-\varphi$ to the manufacturer. The retailer makes a take-it-or-leave-it wholesale-price offer to the third-party producer with probability φ (in which case he offers the latter's marginal cost c_S) and the third-party producer makes an offer with probability $1-\varphi$ (in which case he offers $w_S > c_S$, to be determined shortly). In either case, the maximum quantity the retailer can order is K_S. We make the assumption that the national brand has more bargaining power in setting its wholesale price than the third-party producer, and we normalize it to 1. This assumption is plausible given that national brands tend to have loyal customer bases and thus retain greater power in setting their wholesale prices.

As the national brand does not directly observe the outcome of the negotiation between the retailer and the third party, it sets its wholesale price based on the expected outcome of this negotiation, while the store-brand producer and the retailer best respond to the national brand's (new) pricing strategy. We focus on the $3P^N$ setting in this section; the analysis for the $3P^S$ case is similar.

Retailer's Problem

Let (p_N^0, p_S^0) and (p_N^1, p_S^1) denote the retail prices for the national-brand and store-brand products when the retailer loses and wins the negotiation, respectively. Let $3P_\varphi^N$ denote the (new) $3P^N$ setting given the retailer's bargaining power φ. The retailer's problem is to choose prices to maximize:

$$\begin{aligned} \max_{p_N^0, p_S^0, p_N^1, p_S^1} &= \varphi \left[\left(p_N^1 - w_N\right) \cdot D_N\left(p_N^1, p_S^1\right) + \left(p_S^1 - c_S\right) \cdot D_S\left(p_N^1, p_S^1\right) \right] \qquad (10) \\ &+ (1-\varphi) \left[\left(p_N^0 - w_N\right) \cdot D_N\left(p_N^0, p_S^0\right) + \left(p_S^0 - w_S^0\right) \cdot D_S\left(p_N^0, p_S^0\right) \right] \\ \text{subject to:} \quad & D_S\left(p_N^1, p_S^1\right) \le K_S \qquad (11) \end{aligned}$$

The first (second) term represents the retailer's profit conditional on the retailer (third party) winning the negotiation, multiplied by the respective winning (losing) probability.

National Brand's Problem

Although the national brand does not observe the outcome of the retailer's negotiation with the third party when determining his wholesale price, we assume that the retailer's relative bargaining position is common knowledge. Given the national brand's beliefs about the outcome of the negotiation, his objective is to choose w_N to maximize his expected profit:

$$\max_{w_N} \Pi_N(w_N) = \varphi \left[(w_N - c_N) D_N\left(p_N^1, p_S^1\right) \right] + (1-\varphi) \left[(w_N - c_N) D_N\left(p_N^0, p_S^0\right) \right] \qquad (12)$$

Third-Party Producer's Problem

We first note that the third party will have zero profit if the negotiation favors the retailer. With probability $(1-\varphi)$, the negotiation favors the third party, allowing it to set a wholesale price of its choosing. The third party then chooses a wholesale price w_S^0 to maximize:

$$\begin{aligned} \max_{w_S^0} \Pi_S\left(w_S^0, \varphi\right) &= (1-\varphi) \left[(w_S^0 - c_S) D_S\left(p_N^0, p_S^0\right) \right] \qquad (13) \\ \text{subject to:} \quad & D_S\left(p_N^0, p_S^0\right) \le K_S \end{aligned}$$

There are three possible outcomes vis-à-vis capacity utilization of the store-brand factory in equilibrium: (i) the store-brand factory is unconstrained whether the negotiation outcome favors the retailer or the third-party (denoted as the UU equilibrium); (ii) the store-brand factory is constrained when the retailer wins the negotiation, but unconstrained if the third-party wins the negotiation (denoted by CU); and (iii) the store-brand factory is constrained in equilibrium independent of who has the upper hand in the negotiation (denoted by CC). The UC case, in which the store-brand factory is unconstrained if the retailer wins the negotiation and constrained if the third party wins, never occurs in equilibrium. This is because if the retailer wins the negotiation, double marginalization does not occur, whereupon the retailer would always choose to produce at least as much as an independent third party. The following result describes our findings. The proof is omitted in the interest of brevity because the results are intuitive. Details are available from the authors.

Proposition 4 *The following hold in equilibrium:*
(i) Over the parameter range where the retailer prefers selling the factory in the base model, both the retailer's expected profit and benefit from selling the factory are decreasing in its bargaining power φ; for all other specifications, the retailer does not sell the factory and his profit is unchanged.
(ii) For bargaining power $\varphi \in (0,1)$, the retailer's expected profit under $3P_{\varphi}^{N}$ is between his profit under the $3P^N$ ($\varphi = 0$) and IP ($\varphi = 1$) outcomes in the base model.

From the above, it follows that as the retailer's bargaining power, φ, increases, the *difference* between the retailer's expected profit under IP and $3P_{\varphi}^{N}$ diminishes. Hence, the retailer's incentive to sell the factory declines. This is intuitive, since with $\varphi > 0$, the retailer does not completely relinquish control over wholesale pricing of the store-brand product to the third-party producer. The national brand manufacturer, in turn, retains some of his first-mover advantage, which diminishes the retailer's benefit from selling the factory.

8 Conclusion

There are many non-financial reasons for why a retailer may choose to retain its store-brand factories, such as better control over product quality and delivery reliability. There are also non-financial reasons for why a retailer may choose to sell its factories, including union-related issues and a desire to focus on its core competencies. In this paper, we focus on the financial consequences that arise due to changes in the competitive environment and examine some of the implications of the retailer selling his factory to a third party.

We study a model in which a store-brand product is produced at a retailer's capacity-limited factory and competes with a national brand. For settings with third-party production of the store-brand product, we considered several plausible power relationships among the parties which we capture in the form of different pricing game structures, including one in which there is (wholesale) price negotiation between the retailer and the third-party producer of the store brand product.

We characterize how the equilibrium price offered by the national brand as a function of the store-brand's capacity changes if the store-brand factory is sold to a third-party. Although one may expect the equilibrium wholesale price offered by the national brand to increase once the retailer's factory is sold, we find that capacity constraints have unintuitive effects. In particular, in cases where the store-brand factory is operating at capacity in equilibrium, the national brand's wholesale price may be lower when a third party operates the store-brand factory. Furthermore, due to lower national-brand wholesale prices and the concomitant effects on equilibrium, the retailer may be better off selling his factory. We provide comparative statics on equilibrium decisions and outcomes, including prices, demands, and profits, as a function of the capacity limitation of the store-brand factory.

Our findings indicate that the answer to the question of whether the retailer should sell his factory is not straightforward — it depends on the competitive relationship between the national brand and a third-party owner of the store-brand factory, and it depends intricately on the capacity limitation of the store-brand factory, Our analysis does not take into account any of the retailer's fixed costs of ownership and operation of the factory, but only variable production costs; thus, if such fixed costs are high, the retailer's incentives to sell the factory would be higher than those we describe.

Possible avenues for further research include exploring the impact of different types of demand functions, including those based on more detailed models of customer choice, incorporating additional cases of information asymmetry, considering competition at the retail level, and allowing for dynamic elements such as repeated interactions. As we mentioned earlier, there are settings in which the retailer has more market power than one or both of the manufacturers. In addition, some third-party producers of store-brand products have become larger and more powerful in recent years, often producing store brands for multiple retailers. In some cases, these third-party producers are more powerful than the retailers to whom they sell, and may even be more powerful than regional or weak national brands. Moreover, national-brand manufacturers have become more prominent players in the production of store brands (Kumar and Steenkamp, 2007). In some product categories, there is more than one major national brand. Thus, a multitude of different supply-chain ownership and power structures arise in the competitive marketplace involving store brands and national brands. Further research is needed to understand how supply chain structures and power relationships, as well as operational factors such as capacity constraints, affect equilibrium prices and profits in these settings.

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Appendix A Retailer's Optimal Prices

In the case of internal production, the retailer's reaction function given the national brand's wholesale price, w_N, and the store-brand factory's production cost, c_S, can be derived from the first order conditions of (1):

$$\begin{aligned} \frac{\partial \Pi_R}{\partial p_N} &= \alpha_N + (\beta_N + \gamma)w_N + 2\gamma p_S - \gamma c_S - 2(\beta_N + \gamma)p_N = 0 \\ \frac{\partial \Pi_R}{\partial p_S} &= \alpha_S + (\beta_S + \gamma)c_S + 2\gamma p_N - \gamma w_N - 2(\beta_S + \gamma)p_S = 0 \end{aligned} \tag{14}$$

When the retailer's factory's capacity constraint is binding, the first order conditions resulting from the Lagrangian objective tied to (1) are:

$$\frac{\partial L_R}{\partial p_N} = \alpha_N + (\beta_N + \gamma)w_N + 2\gamma p_S - \gamma c_S - 2(\beta_N + \gamma)p_N - M\gamma = 0 \tag{15}$$

$$\frac{\partial L_R}{\partial p_S} = \alpha_S + (\beta_S + \gamma)c_S + 2\gamma p_N - \gamma w_N - 2(\beta_S + \gamma)p_S + M(\beta_S + \gamma) = 0 \tag{16}$$

$$\frac{\partial L_R}{\partial M} = K_S - (\alpha_S - (\beta_S + \gamma)p_S + \gamma p_N) = 0 \tag{17}$$

where M is the Lagrange multiplier. From the expressions above, we can show that when the retailer's capacity constraint is not binding, p_N^* and p_S^* can be expressed as: $p_N^* = \frac{\alpha}{2\beta} + 0.5w_N$ and $p_S^* = \frac{\alpha}{2\beta} + 0.5c_S$. On the other hand, if the retailer's factory's capacity constraint is binding, we have:

$$p_N^* = \frac{\alpha_N(\beta_S + \gamma) + \gamma\alpha_S}{2\left(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma\right)} + 0.5w_N \quad \text{and} \quad p_S^* = \frac{\alpha_S(\beta_N + \gamma) + \gamma\alpha_N}{2\left(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma\right)} + 0.5c_S$$

Appendix B Equilibrium Wholesale Prices and Profits — Nash Competition ($3P^N$)

The manufacturers' equilibrium wholesale prices can be derived from the following first-order conditions of the respective manufacturers' profit maximization problems: $\Pi_{i,3P^N}$ with respect to w_i, $i \neq j$ is:

$$\frac{\partial \Pi_{i,3P^N}}{\partial w_i} = -(\beta_i+\gamma)w_i + \frac{\alpha_i + \gamma w_j + c_i(\beta_i+\gamma)}{2} \tag{18}$$

Setting the derivative equal to zero, we get the optimal response functions for suppliers i and j, $w_i^*(w_j)$ and $w_j^*(w_i)$:

$$w_i^*(w_j) = \frac{\alpha_i}{2(\beta_i+\gamma)} + \frac{\gamma w_j}{2(\beta_i+\gamma)} + 0.5c_i \quad \text{and} \quad w_j^*(w_i) = \frac{\alpha_j}{2(\beta_j+\gamma)} + \frac{\gamma w_i}{2(\beta_j+\gamma)} + 0.5c_j$$

Solving the above equations simultaneously, we find the equilibrium wholesale prices under the unconstrained $3P^N$ configuration to be:

$$w_{i,3P^N}^{U^*} = \frac{\gamma\alpha_i + (\beta_i+\gamma)[2\alpha_j + \gamma c_i + 2(\beta_j+\gamma)c_j]}{4(\beta_j+\gamma)(\beta_i+\gamma)-\gamma^2} \quad \text{and} \quad w_{j,3P^N}^{U^*} = \frac{\gamma\alpha_j + (\beta_j+\gamma)[2\alpha_i + \gamma c_j + 2(\beta_i+\gamma)c_i]}{4(\beta_i+\gamma)(\beta_j+\gamma)-\gamma^2}$$

Substituting these equilibrium wholesale prices into (8), we find that the national brand's equilibrium profit, under the $3P^N$ configuration when the store-brand factory is capacity unconstrained is:

$$\Pi_{N,3P^N}^{U^*} = \frac{(\beta_N+\gamma)\left[\gamma\alpha_S + 2(\beta_S+\gamma)\alpha_N + \gamma(\beta_S+\gamma)c_S - \left[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]c_N\right]^2}{2\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2} \tag{19}$$

We next find the equilibrium wholesale prices and national brand's profits when the third-party producer is capacity constrained. The third party supplier is capacity constrained if

$$\alpha_S - (\beta_S+\gamma)\left[\frac{\alpha_S(\beta_N+\gamma)+\gamma\alpha_N}{2(\beta_N\beta_S+\beta_N\gamma+\beta_s\gamma)} + \frac{1}{2}w_S\right] + \gamma\left[\frac{\alpha_N(\beta_S+\gamma)+\gamma\alpha_S}{2(\beta_N\beta_S+\beta_N\gamma+\beta_s\gamma)} + \frac{1}{2}w_N\right] \geq K_S$$

Thus, when the factory is capacity constrained, the capacity-clearing price is

$$w_S^*(w_N) = \frac{\alpha_S - 2K_S + \gamma w_N}{(\beta_S+\gamma)}. \tag{20}$$

Simultaneously solving (20) along with the national brand's wholesale price response, $w_N^*(w_S) = \frac{\alpha_N}{2(\beta_N+\gamma)} + \frac{\gamma w_S}{2(\beta_N+\gamma)} + 0.5c_N$, we obtain the equilibrium wholesale price for the store brand and national brand supplier: $w_{S,3P^N}^{C^*} = \frac{\gamma\alpha_N + (\beta_N+\gamma)(2\alpha_S + c_N\gamma - 4K_S)}{2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2}$ and $w_{N,3P^N}^{C^*} = \frac{\gamma(\alpha_S-2K_S) + (\beta_S+\gamma)[\alpha_N + (\beta_N+\gamma)c_N]}{2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2}$, respectively.

Substituting both $w_{N,3P^N}^{C^*}$ and $w_{S,3P^N}^{C^*}$ into (8), we find that the national brand's equilibrium profit under the $3P^N$ configuration when the third party supplier is capacity constrained to be:

$$\Pi_{N,3P^N}^{C^*} = \frac{(\beta_N+\gamma)\left[(\beta_S+\gamma)\alpha_N - ((\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)c_N + \gamma(\alpha_S-2K_S)\right]^2}{2\left[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2} \tag{21}$$

Appendix C Equilibrium Wholesale Prices and Profits — Stackelberg Configuration ($3P^S$)

C.1 Third Party Supplier Problem – Unconstrained Store-Brand Factory

For any value of w_N, the third party supplier's wholesale price response can be derived from the following first-order condition tied to his profit-maximization problem:

$$\frac{\partial \Pi_S}{\partial w_S} = -(\beta_S+\gamma)w_S + \frac{\alpha_S}{2} + \frac{1}{2}\gamma w_N + \frac{1}{2}c_S\left(\beta_S+\gamma\right) \tag{22}$$

Solving for w_S, we obtain the third-party supplier's optimal response:

$$w_S^*(w_N) = \frac{\alpha_S}{2(\beta_S+\gamma)} + \frac{\gamma w_N}{2(\beta_S+\gamma)} + 0.5c_S \tag{23}$$

C.2 National Brand Supplier Problem

Using the third-party supplier's optimal wholesale price response function, the national brand's wholesale price response can be derived from the following first-order condition tied to his profit-maximization problem:

$$\frac{\partial \Pi_{N,3P^S}}{\partial w_N} = -\frac{2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2}{2(\beta_S+\gamma)}w_N + \frac{\alpha_N}{2} + \frac{\gamma\alpha_S}{4(\beta_S+\gamma)} + \frac{\gamma c_S}{2} + \frac{2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2}{4(\beta_S+\gamma)}c_N \tag{24}$$

Solving (24) for w_N, we have $w_{N,3P^S}^{U^*} = \frac{c_N}{2} + \frac{\gamma\alpha_S + (\beta_S+\gamma)\left[2\alpha_N+\gamma c_S\right]}{4(\beta_N+\gamma)(\beta_S+\gamma)-2\gamma^2}$. Substituting $w_{N,3P^S}^{U^*}$ into (23), we find the equilibrium wholesale price for the third party supplier to be

$$w_{S,3P^S}^{U^*} = \frac{\gamma c_N}{4(\beta_S+\gamma)} + \frac{2(\beta_S+\gamma)\gamma\alpha_N + \left(4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right)\left[\alpha_S+(\beta_S+\gamma)c_S\right]}{4[(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)(\beta_S+\gamma)]}$$

Substituting for the equilibrium wholesale prices into the national brand's profit function, we find that the national brand's equilibrium profit under this capacity configuration is:

$$\Pi_{N,3P^S}^{U^*} = \frac{1}{16}\left[\frac{\left[2(\beta_S+\gamma)\alpha_N + \gamma(\alpha_S+(\beta_S+\gamma)c_S) - (2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)c_N\right]^2}{(\beta_S+\gamma)(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)}\right] \tag{25}$$

C.3 Third-Party Supplier Problem – Capacity Constrained Store-Brand Factory

As given by (20), the market-clearing value of w_S when the factory is capacity constrained is $w_S^*(w_N) = \frac{\alpha_S - 2K_S + \gamma w_N}{(\beta_S+\gamma)}$. Using (2) and the third-party supplier's optimal wholesale price response above, the national brand's wholesale price response can be derived from the following first-order condition:

$$\frac{\partial \Pi_{N,3P^S}}{\partial w_N} = -\frac{\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma}{2(\beta_S+\gamma)}w_N + \frac{\alpha_N}{2} + \frac{\gamma\alpha_S}{2(\beta_S+\gamma)} - \frac{\gamma 2K_S}{2(\beta_S+\gamma)} \tag{26}$$

Solving (26) for w_N, we find that the national brand's equilibrium wholesale price is

$$w_{N,3P^S}^{C^*} = \frac{c_N}{2} + \frac{\gamma(\alpha_S-2K_S) + (\beta_S+\gamma)\alpha_N}{2(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} = w_{N,IP}^{C^*}$$

Substituting $w_{N,3P^S}^{C^*}$ into the expression for the store-brand supplier's optimal response, $w_S^*(w_N)$ above, we find the equilibrium wholesale price for the third-party supplier to be

$$w_{S,3P^S}^{C^*} = \frac{\gamma c_N}{2(\beta_S+\gamma)} + \frac{[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2](\alpha_S-2K_S) + \gamma(\beta_S+\gamma)\alpha_N}{2(\beta_S+\gamma)[\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma]}$$

Using the expressions for $w_{N,3P^S}^{C^*}$ and $w_{S,3P^S}^{C^*}$, we find the national brand's equilibrium profit under this capacity configuration to be:

$$\Pi^{C^*}_{N,3P^S} = \frac{1}{8}\left[\frac{[(\beta_S+\gamma)\alpha_N + \gamma(\alpha_S - 2K_S) - (\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)c_N]^2}{(\beta_S+\gamma)(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)}\right] \tag{27}$$

Appendix D Proof of Relationships Among Capacity Thresholds

In this section, we show that $K_S^{3P^N} \leq K_S^{3P^S} \leq K_S^{IP}$.

D.1 Proof of $K_S^{3P^N} \leq K_S^{3P^S}$:

Recall that $K_S^{3P^N}$ and $K_S^{3P^S}$ are given by (9) and (7), respectively. Under the *3P*^{S} configuration, the contribution margin for the store-brand supplier can be written as:

$$w^{U^*}_{S,3P^S} - c_S = \frac{\gamma c_N}{4(\beta_S+\gamma)} + \frac{(4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)\,[\alpha_S + (\beta_S+\gamma)c_S] + 2(\beta_S+\gamma)\gamma\alpha_N}{4[(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)(\beta_S+\gamma)]} - c_S$$

which must be nonnegative. Thus, c_S must satisfy the following, where the right hand side is an upper bound on c_S:

$$c_S \leq \frac{2(\beta_S+\gamma)\gamma\alpha_N + \alpha_S[(4\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2] + [2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]c_N}{(\beta_S+\gamma)(4(\beta_N+\gamma)(\beta_S+\gamma)-3\gamma^2)} \tag{28}$$

Substituting the upper bound for c_S into the difference of (9) and (7), we obtain the following expression as a lower bound on $K_S^{3P^S} - K_S^{3P^N}$:

$$K_S^{3P^S} - K_S^{3P^N} \geq \frac{2[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]\cdot C}{[4(\beta_N+\gamma)(\beta_S+\gamma)-3\gamma^2]}\left[\frac{(\beta_S+\gamma)\alpha_N + \gamma\alpha_S - (\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)c_N}{\gamma}\right] > 0$$

where $C = \frac{2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2}{4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2} + \sqrt{\frac{(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)}{2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2}} > 0$. Therefore, $K_S^{3P^S} - K_S^{3P^N} \geq 0$, or $K_S^{3P^S} \geq K_S^{3P^N}$.

D.2 Proof of $K_S^{3P^S} \leq K_S^{IP}$:

Using (7), we have $K_S^{3P^S} \leq K_S^{IP}$ if

$$\sqrt{\frac{[2(\beta_S+\gamma)\alpha_N+\gamma(\alpha_S+(\beta_S+\gamma)c_S)-(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)c_N]^2}{2[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]}} \geq \sqrt{\frac{(\beta_S+\gamma)(\alpha_N-(\beta_N+\gamma)c_N+\gamma c_S)^2}{(\beta_N+\gamma)}}$$

Moving the squared terms outside the square root, and subtracting the righthand side from the lefthand side, we get:

$$\underbrace{\sqrt{\frac{1}{2\,[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]}}}_{\geq 0}\cdot\gamma\,[\alpha_S-(\beta_S+\gamma)c_S+\gamma c_N] + \underbrace{\left[\sqrt{\frac{4(\beta_S+\gamma)^2}{2\,[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]}} - \sqrt{\frac{(\beta_S+\gamma)}{(\beta_N+\gamma)}}\right]}_{\geq 0} \cdot[\alpha_N-(\beta_N+\gamma)c_N+\gamma c_S] \geq 0$$

The latter part of each term is just the store- and national-brand demand, respectively, at (c_N, c_S), which are non-negative. Thus, $K_S^{3P^S} \leq K_S^{IP}$ indeed holds.

Appendix E Proof of Results in Figure 2

In this appendix, we detail the relationship among the wholesale prices under *IP*, *3P*^{N} and *3P*^{S} in the different capacity regions using four propositions. For each region, we express the relationships among the wholesale prices in the statement of the proposition, followed by its proof.

Proposition A *For K_S in Region 1, where the store-brand factory's capacity constraint is binding under IP, 3P^N, and 3P^S, the national brand's equilibrium wholesale prices satisfy* $w^{C^*}_{N,3P^N} < w^{C^*}_{N,IP} = w^{C^*}_{N,3P^S}$.

Proof: Using algebraic manipulations, we can show that the condition $w^{C^*}_{N,IP} > w^{C^*}_{N,3P^N}$ can be written as:

$$\frac{\gamma(\alpha_S - 2K_S) + (\beta_S + \gamma)\alpha_N}{2(\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)} > 0.5c_N \tag{29}$$

Notice that the left-hand side of the above inequality is equal to the first term in $w^{C^*}_{N,IP}$ for the capacity-constrained case (see Table 1). Therefore, if the above inequality is not satisfied, then $w^{C^*}_{N,IP}$ would not exceed c_N and the national brand would not participate. Thus, we have established that $w^{C^*}_{N,IP} > w^{C^*}_{N,3P^N}$. As shown in the analysis of the *3P^S* configuration (in Appendix C.3), $w^{C^*}_{N,IP} = w^{C^*}_{N,3P^S}$. Thus, $w^{C^*}_{N,IP} = w^{C^*}_{N,3P^S} > w^{C^*}_{N,3P^N}$. This completes the proof. ■

Given the above, we consider the relationship between $w^{C^*}_{N,3P^N}$ and $w^{C^*}_{N,IP}$. If the store-brand factory has a binding capacity constraint under *3P^N* with the third-party charging a mark-up over production cost, then the store-brand factory faces even tighter capacity restrictions in the absence of a wholesale mark-up. To compensate, the retailer has to pass on a high opportunity cost of capacity to customers to drive down demand. Facing little competition, the national brand could charge a high wholesale price. With the introduction of the third-party "middleman" under *3P^N*, the store-brand factory faces less problematic capacity restrictions, which then forces the national brand to price more competitively under *3P^N* than under IP.

We now discuss why $w^{C^*}_{N,IP} = w^{C^*}_{N,3P^S}$, which is a more subtle result. In Region 1, the store-brand factory is operating at capacity under both *3P^S* and IP and the store-brand factory provides the same supply to the market whether the factory is operated by the retailer or by a third party. In the IP configuration, the national brand chooses his wholesale price so that the resultant demand for the store-brand product is exactly equal to the factory's capacity. Under *3P^S*, the eventual outcome is the same, but the third-party manufacturer performs the capacity-clearing function (to ensure that his capacity constraint is not violated) via his choice of wholesale price in response to the national brand's wholesale price and in view of how the retailer will choose retail prices. In both cases, the national brand uses his first-mover advantage to "push" the store-brand factory to operate at full capacity. This turns out to be preferable for the national brand: although he sells fewer units, he sells each unit at a higher margin, and the differential in the margin more than compensates for the reduction in sales volume.

The same result as in Proposition A applies, with a similar rationale, when K_S is in Region 2, as stated in the next proposition.

Proposition B *For K_S in Region 2, where the store-brand factory is constrained under IP and 3P^S but unconstrained under 3P^N, the national brand's equilibrium wholesale prices satisfy* $w^{U^*}_{N,3P^N} < w^{C^*}_{N,IP} = w^{C^*}_{N,3P^S}$.

Proof: At the leftmost boundary of Region 2, $w^{C^*}_{N,3P^N} < w^{C^*}_{N,IP} = w^{C^*}_{N,3P^S}$ (by Proposition A). At the rightmost boundary, the store-brand factory is unconstrained under both *3P^N* and *3P^S*. Algebraically we can show that the condition $w^{U^*}_{N,3P^S} > w^{U^*}_{N,3P^N}$ can be written as $\dfrac{\gamma\alpha_S + (\beta_S+\gamma)(2\alpha_N + \gamma c_S)}{4(\beta_N+\gamma)(\beta_S+\gamma) - 2\gamma^2} > 0.5c_N$.

Using a similar argument to that in the proof of Proposition A, we note that the left hand side of the above inequality is equal to the first term in $w^{U^*}_{N,3P^S}$. Therefore, if the above inequality is not satisfied, then the national brand would not participate. Thus, we have established that $w^{U^*}_{N,3P^S} > w^{U^*}_{N,3P^N}$ in Region 3 (and at the rightmost boundary of Region 2). The facts that in Region 2, $w^{U^*}_{N,3P^N}$ is constant and $w^{C^*}_{N,3P^S}$ is linearly decreasing in K_S complete the proof. ■

Proposition C *For K_S in Region 3, where the store-brand factory is constrained under IP but unconstrained under 3P^N and 3P^S, the national brand's equilibrium wholesale prices in the two outsourcing configurations are such that $w^{U^*}_{N,3P^N} < w^{U^*}_{N,3P^S}$. Furthermore, the wholesale price under IP, $w^{C^*}_{N,IP}$, may be larger, smaller,* *or in-between $w^{U^*}_{N,3P^N}$ and $w^{U^*}_{N,3P^S}$, depending upon the capacity level of the store-brand factory.*

Proof: See proof of Proposition B above. ■

Recall that the national brand's wholesale price is the same under *IP* and *3P*^{S} in Regions 1 and 2. At the lower boundary of Region 3, the store-brand factory becomes unconstrained under *3P*^{S}. Hence, the national brand's price becomes constant under *3P*^{S} while it continues to fall under *IP* because the store-brand factory is still constrained under *IP* in this region. Thus, the relationship between these two curves is not surprising. What is more interesting is that the price curve under *IP* crosses that of *3P*^{N} in this region. For smaller values of K_S within Region 3, the binding capacity of the store-brand factory under *IP* combined with the national brand's first-mover advantage enables the national brand to charge a high price despite the absence of double marginalization on the store-brand product. On the other hand, as K_S increases, the national brand must be more price-competitive due to greater supply of the store-brand product. Indeed, despite the national brand's first mover advantage under *IP*, when there is near-ample capacity at the store-brand factory, the national brand cannot charge as much as he could have under *3P*^{N}. This occurs because in the latter scenario, the national brand benefits from double marginalization on the store brand product, but such "protection" does not exist when the retailer owns the factory.

Proposition D *For K_S in Region 4, where the store-brand factory is unconstrained under all supply chain configurations, the national brand's equilibrium wholesale prices are such that $w^{U^*}_{N,IP} < w^{U^*}_{N,3P^N} < w^{U^*}_{N,3P^S}$.*

Proof: The proof is straightforward and is omitted.

Appendix F Proof of Results in Section 5

F.1 Proof of Lemma 1:

Proof of part (a): For K_S in Region 1, we can show that the first derivative of $\Pi^{C^*}_{R,3P^N}$ with respect to K_S is strictly greater than zero, and the second derivative is non-negative. For Region 2, 3 and 4, where the factory is unconstrained under $3P^N$, $\Pi^*_{R,3P^N}$ does not depend on K_S. Thus, the retailer's profit function is convex increasing in K_S for Region 1, and constant for Regions 2, 3, and 4.

Proof of part (b): The retailer's profit in Regions 1, 2, and 3, under IP, is concave increasing in K_S. For K_S in Region 4, the profit function is constant. To show there a jump discontinuity at K_S^{IP}, we consider $\Pi^{U^*}_{R,IP} - \Pi^{C^*}_{R,IP}\left(K_S^{IP}\right)$, and show this difference is strictly greater than zero. The retailer's profit function under IP can be found in Table 1. We can show that the difference between the unconstrained profit and the constrained profit at K_S^{IP}, $\Pi^{U^*}_{R,IP} - \Pi^{C^*}_{R,IP}\left(K_S^{IP}\right) > 0$. Thus, we have a jump discontinuity at $K_S^{3P^S}$.

Proof of part (c): For K_S in Regions 1 and 2, the retailer's profit function is convex increasing in K_S. For Region 3 and 4, where the factory is unconstrained under $3P^S$, $\Pi^{U^*}_{R,3P^S}$ does not depend on K_S, and, therefore, is constant. Thus, the retailer's profit function is convex increasing in K_S for Region 1 and 2, and constant for Regions 3 and 4. The discontinuity in the retailer's functions at $K_S^{3P^S}$ arises because the retailer sells more national brand product at a strictly higher margin, and sells the store-brand product at a sufficiently higher margin to offset a decrease in store-brand product demand. In other words, the profits from selling both products are strictly larger. The derivation details are omitted due to space considerations.

F.2 Proof of Proposition 1

We begin by showing that the retailer's profit under *3P*^{N} exceeds his profits under IP at $K_S = 0$. At $K_S = 0$, the retailer's profit under the IP and *3P*^{N} scenarios are, respectively:

$$\Pi^{C^*}_{R,IP}(K_S = 0) = \frac{\left[\gamma\alpha_S + (\beta_S+\gamma)\alpha_N - (\beta_N\beta_S + \beta_N\gamma + \beta_S\,\gamma)c_N\right]^2}{16(\beta_S+\gamma)(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} \tag{30}$$

$$\Pi^{C^*}_{R,3P^N}(K_S = 0) = \frac{(\beta_N+\gamma)^2(\beta_S+\gamma)\left[\gamma\alpha_S + (\beta_S+\gamma)\alpha_N - (\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)c_N\right]^2}{4\left[(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)\left(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right)^2\right]} \tag{31}$$

We note that

$$\Pi^{C^*}_{R,3P^N}(K_S=0) \quad > \quad \frac{(\beta_N+\gamma)^2(\beta_S+\gamma)\left[\gamma\alpha_S+(\beta_S+\gamma)\alpha_N-(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)c_N\right]^2}{4\left[(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)\left(2(\beta_N+\gamma)(\beta_S+\gamma)\right)^2\right]} > \Pi^{C^*}_{R,IP}(K_S=0)$$

Thus, at $K_S = 0$, the retailer's profit under *3P*^{N} exceeds that under IP. Because $\Pi^*_{R,3P^N}$ is convex increasing in K_S, and $\Pi^*_{R,IP}$ is concave increasing in Region 1, the two functions cross at most twice in Region 1.

F.3 Proof of Corollary 1

In Region 2, the retailer's profit under *3P*^{N} is constant, and is increasing under IP. Thus, they may cross at most once in Region 2. If the retailer is better off under *3P*^{N} at $K_S^{3P^N}$, then the retailer will be better off for some K_S-interval in Region 2.

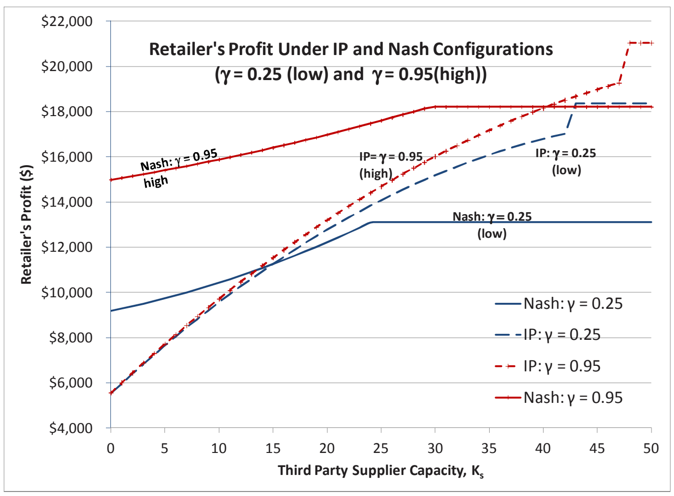

Figure 8: Equilibrium configurations under IP, *3P*^{N}, and *3P*^{S} as a function of K_S.

F.4 The Effect of Substitutability on the Retailer's Profits

We consider an example with $\alpha_N = 100$, $\alpha_S = 50$, $c_N = c_S = 1$, $\beta = 0.15$, two degrees of substitutability: $\gamma = 0.25$ (low) and $\gamma = 0.95$ (high). In Figure 8, we show the retailer's profit as a function of K_S for the two values of γ under both the IP and $3P^N$ configurations. For the high value of γ, we observe, as expected, that the retailer prefers the Nash configuration over a broad range of K_S values. For the low value of γ, we observe that the retailer prefers the Nash configuration over a smaller range of K_S values. Yet, the resulting market share for the store brand product (with $\gamma = 0.25$) is close to 25%, which is consistent with market shares observed in grocery retailing (Geller (2011)). Although the retailer achieves higher profit when γ is high, low substitutability does not preclude the retailer from benefiting from the sale of its store-brand factory. This is because even for low γ values, the effect of competition can dominate the effect of double-marginalization.

Appendix G Results Regarding National Brand's Profits

G.1 Proof of Lemma 2

Recall that the national brand's profits under the IP configuration are given in Table 1. The first and second derivatives of $\Pi^*_{N,IP}$ with respect to K_S are:

$$\frac{\partial \Pi^*_{N,IP}}{\partial K_S} = \begin{cases} \dfrac{\gamma[-(\beta_S+\gamma)\alpha_N + (\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)c_N - \gamma(\alpha_S - 2K_S)]}{2(\beta_S+\gamma)(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} < 0 & \text{for } K_S \in [0, K_S^{IP}] \\ 0 & \text{for } K_S \in [K_S^{IP}, \infty) \end{cases}$$

$$\frac{\partial^2 \Pi^*_{N,IP}}{\partial K_S^2} = \begin{cases} \dfrac{\gamma^2}{(\beta_S+\gamma)(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} > 0 & \text{for } K_S \in [0, K_S^{IP}] \\ 0 & \text{for } K_S \in [K_S^{IP}, \infty) \end{cases}$$

Substituting the expression for K_S^{IP} in $\Pi^{*C}_{N,IP}$, we find that all the terms not in the square root cancel, and we are left with

$$\Pi^{*C}_{N,IP}(K_S^{IP}) = \frac{\left[\sqrt{\frac{(\beta_S+\gamma)(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)(\alpha_N-(\beta_N+\gamma)c_N+\gamma c_S)^2}{(\beta_N+\gamma)}}\right]^2}{8(\beta_S+\gamma)(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)} = \frac{(\alpha_N-(\beta_N+\gamma)c_N+\gamma c_S)^2}{8(\beta_N+\gamma)}$$

$$= \Pi^{*U}_{N,IP}(K_S^{IP})$$

The above results imply that the national brand's profit function under the IP configuration is continuous, convex decreasing in K_S, $K_S \in [0, K_S^{IP}]$, and constant for $K_S \geq K_S^{IP}$.

For the *3P*^{S} configuration, we showed in Section 3.2.2 that $\Pi^{*C}_{N,3P^S} = \Pi^{*C}_{N,IP}$ for $K_S \in [0, K_S^{3P^S}]$. Thus, by the preceding analysis, we know the national brand's profit is convex decreasing in K_S for $K_S \in [0, K_S^{3P^S}]$, and constant for $K_S \geq K_S^{3P^S}$. The national brand's profit, given by (25), is:

$$\Pi^{C*}_{N,3P^S}(K_S = K_S^{3P^S}) = \left[\frac{\left[(\beta_S+\gamma)\alpha_N + \gamma\left(\alpha_S - 2K_S^{3P^S}\right) - (\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)c_N\right]^2}{8(\beta_S+\gamma)(\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)}\right]$$

$$= \frac{1}{16}\left[\frac{\left[2(\beta_S+\gamma)\alpha_N + \gamma(\alpha_S+(\beta_S+\gamma)c_S) - (2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)c_N\right]^2}{(\beta_S+\gamma)(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)}\right] = \Pi^{U*}_{N,3P^S}\left(K_S^{3P^S}\right)$$

which is a constant. Thus, in summary, the national brand's profit function under the *3P*^{S} configuration is continuous, convex decreasing in K_S, $K_S \in [0, K_S^{3P^S}]$, and constant for $K_S \geq K_S^{3P^S}$.
The national brand supplier's profit functions under the *3P*^{N} configuration are given by (19) and (21) in the unconstrained and constrained cases, respectively. The first and second derivatives of (19) and (21) with respect to K_S are:

$$\frac{\partial \Pi^*_{N,3P^N}}{\partial K_S} = \begin{cases} \dfrac{2\gamma(\beta_N+\gamma)[-(\beta_S+\gamma)\alpha_N + (\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma)c_N - \gamma(\alpha_S-2K_S)]}{[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]^2} < 0, & K_S \in [0, K_S^{3P^N}] \\ 0, & K_S \in [K_S^{3P^N}, \infty) \end{cases}$$

$$\frac{\partial^2 \Pi^*_{N,3P^N}}{\partial K_S^2} = \begin{cases} \dfrac{4\gamma^2(\beta_N+\gamma)}{[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2]^2} > 0 & \text{for } K_S \in [0, K_S^{3P^N}] \\ 0 & \text{for } K_S \in [K_S^{3P^N}, \infty) \end{cases}$$

Although it is algebraically messy, it is straightforward to show that $\Pi^{C*}_{N,3P^N}(K_S^{3P^N}) = \Pi^{U*}_{N,3P^N}(K_S^{3P^N})$. Hence, the national brand's profit function under the *3P*^{N} configuration is continuous, convex decreasing in K_S, $K_S \in [0, K_S^{3P^N}]$, and constant for $K_S \geq K_S^{3P^N}$.

G.2 Proof of Lemma 3

We now compare the national brand's profits under different configurations, region by region.

We first prove that *for* K_S *in Region 1 (i.e.,* $K_S \in [0, K_S^{3P^N}]$*),* $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S} = \Pi^*_{N,IP}$. In Section 3.2.3, we showed that $\Pi^*_{N,IP} = \Pi^*_{N,3P^S}$ in Region 1, Thus, we only need to show that $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S}$ in Region 1, which we establish next.The national brand's profits under the Nash and Stackelberg settings in Region 1 are given by (21) and (27), respectively. Dividing (27) by (21), we have

$$\frac{\Pi^*_{N,3PS}}{\Pi^*_{N,3PN}} = \frac{\left[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2}{4(\beta_N+\gamma)(\beta_S+\gamma)\left[(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]} \geq 1.$$

Thus, $\Pi^*_{N,3P^N} < \Pi^*_{N,3PS} = \Pi^*_{N,IP}$ for all K_S in Region 1.

We now prove that *for* K_S *in Region 2 (i.e.,* $K_S \in [K_S^{3P^N}, K_S^{3P^S}]$*),* $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S} = \Pi^*_{N,IP}$. In Lemma 4, we established that $\Pi^*_{N,3PN}$ is constant in Region 2. In Section 3.2.3 we showed that $\Pi^*_{N,3PS} = \Pi^*_{N,IP}$ and in Lemma 4, we showed that these functions are decreasing in Region 2. Thus, to show that $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S} = \Pi^*_{N,IP}$ in Region 2, we only need to show that $\Pi^*_{N,3P^N} < \Pi^*_{N,IP}$ at $K_S^{3P^S}$, the rightmost boundary of Region 2, which we establish next.

$$\begin{aligned}
\Pi^*_{N,3P^N}(K_S^{3P^S}) &= \frac{(\beta_N+\gamma)\cdot C_1}{2\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2} = \frac{(\beta_N+\gamma)(\beta_S+\gamma)\cdot C_1}{2(\beta_S+\gamma)\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2} \\
&< \frac{C_1}{4(\beta_S+\gamma)\left[\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma\right]} < \frac{C_1}{16(\beta_S+\gamma)\left[\beta_N\beta_S+\beta_N\gamma+\beta_S\gamma\right]} = \Pi^*_{N,IP}(K_S^{3P^S})
\end{aligned}$$

where $C_1 = \left[\gamma\alpha_S + (\beta_S+\gamma)\left[2\alpha_N+\gamma c_S\right] - \left[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]c_N\right]^2$.

We now prove: *For* K_S *in Region 3 (i.e.,* $K_S \in [K_S^{3P^S}, K_S^{IP}]$*),* $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S}$ *and* $\Pi^*_{N,IP} < \Pi^*_{N,3P^S}$. We have already shown that at $K_S^{3P^S}$ (the leftmost boundary of Region 3), the national brand's profit under IP equals his profit under *3P*^{S}, which exceeds his profit under *3P*^{N}. Because the national brand's profits under both *3P*^{N} and *3P*^{S} are constant in Region 3, it follows that for K_S in Region 3 (i.e., $K_S \in [K_S^{3P^S}, K_S^{IP}]$), $\Pi^*_{N,3P^N} < \Pi^*_{N,3P^S}$. Finally, because the national brand's profit under IP is decreasing in Region 3, and the profit under *3P*^{S} is constant, it must be the case that $\Pi^*_{N,IP} < \Pi^*_{N,3P^S}$ in Region 3.

We now show that *for* K_S *in Region 4 (i.e.,* $K_S \in [K_S^{IP}, \infty]$*),* $\Pi^*_{N,IP} \leq \Pi^*_{N,3P^N} < \Pi^*_{N,3P^S}$. We first prove that $\Pi^*_{N,IP} < \Pi^*_{N,3P^S}$. Because the national brand's profit under IP was shown above to be less or equal to his profit under *3P*^{S} (a constant) at the left boundary of Region 3, and his profit under IP is decreasing in Region 3 while his profit under *3P*^{S} is constant, it must be the case that in Region 4, it remains less than his profit under *3P*^{S}. We next show that $\Pi^*_{N,IP} \leq \Pi^*_{N,3P^N}$. We can write $\Pi^*_{N,IP}$ as:

$$\begin{aligned}
\Pi^*_{N,IP} &= \frac{(\beta_N+\gamma)\left[(\beta_S+\gamma)[2\alpha_N+2\gamma c_S] - \left[2(\beta_N+\gamma)(\beta_S+\gamma)\right]c_N\right]^2}{2\left[4(\beta_N+\gamma)(\beta_S+\gamma)\right]^2} \\
&< \frac{(\beta_N+\gamma)\left[(\beta_S+\gamma)[2\alpha_N+2\gamma c_S] - \left[2(\beta_N+\gamma)(\beta_S+\gamma)\right]c_N\right]^2}{2\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2} \equiv \Pi^{UpperBound}_{N,IP} \qquad (32)
\end{aligned}$$

where $\Pi^{UpperBound}_{N,IP}$ is an upper bound on $\Pi^*_{N,IP}$.

Recall that $\Pi_{N,3P^N}{}^* = \dfrac{(\beta_N+\gamma)\left[\gamma\alpha_S + (\beta_S+\gamma)[2\alpha_N+\gamma c_S] - \left[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]c_N\right]^2}{2\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2}$. Dividing $\Pi_{N,3P^N}{}^*$by (32), we obtain

$$\frac{\Pi_{N,3P^N}{}^*}{\Pi^{UpperBound}_{N,IP}} = \frac{\left[\gamma\alpha_S + (\beta_S+\gamma)[2\alpha_N+\gamma c_S] - \left[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]c_N\right]^2}{\left[(\beta_S+\gamma)[2\alpha_N+2\gamma c_S] - \left[2(\beta_N+\gamma)(\beta_S+\gamma)\right]c_N\right]^2} \geq 1$$

The inequality follows from the fact that the difference between the numerator and denominator is non-negative, i.e., $\left[\gamma\alpha_S + (\beta_S+\gamma)[2\alpha_N + \gamma c_S] - \left[2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2\right] c_N\right] -$

$$[(\beta_S+\gamma)[2\alpha_N + 2\gamma c_S] - [2(\beta_N+\gamma)(\beta_S+\gamma)]\, c_N] = \gamma\left[\alpha_S - (\beta_S+\gamma)c_S + \gamma c_N\right] = \gamma D_S(c_N, c_S) \geq 0 \tag{33}$$

Thus, we have shown that $\Pi^*_{N,IP} < \Pi^{UpperBound}_{N,IP} \leq \Pi^*_{N,3P^N}$ in Region 4.

Appendix H Proof of Lemma 4

It is straightforward to compare the demand expressions given in Tables 1, 2, and the demand expressions in Appendix C at $K_S = 0$ and at the various K_S thresholds to establish the claims in Lemma 4. The expressions for $D^{C^*}_{N,IP}$ and $D^{U^*}_{N,IP}$ can be found in Table 1. Table 2 contains the expressions for $D^{C^*}_{N,3P^N}$ and $D^{U^*}_{N,3P^N}$. Lastly, the expressions for $D^{C^*}_{N,3P^S}$ and $D^{U^*}_{N,3P^S}$ are shown in Appendix C.

For Region 1, we must show that $D^{C^*}_{N,IP} = D^{C^*}_{N,3P^S} \leq D^{C^*}_{N,3P^N}$. Region 1's leftmost boundary is $K_S = 0$. Recall from Tables 1 and 3 that $D^{C^*}_{N,IP} = D^{C^*}_{N,3P^S}$, whereby we only need to establish the inequality between $D^{C^*}_{N,3P^S}$ and $D^{C^*}_{N,3P^N}$. Both demands are linearly decreasing in K_S (cf. Tables 1 and 3). Hence, we need only show that their values at the boundaries of Region 1 have the stated relationship. Substituting $K_S = 0$ into the expression for $D^{C^*}_{N,IP} = D^{C^*}_{N,3P^S}$, through simple algebraic manipulations we find that

$$\begin{aligned} D^{C^*}_{N,IP}(0) &= \frac{(\beta_S+\gamma)\alpha_N - (\beta_N\beta_S + \beta_N\gamma + \beta_S\gamma)c_N + \gamma\alpha_S}{4(\beta_S+\gamma)} \\ &< \frac{(\beta_N+\gamma)\left[(\beta_S+\gamma)\alpha_N - (\beta_N\beta_S + \beta - N\gamma + \beta_S\gamma)c_N + \gamma(\alpha_S - 2K_S)\right]}{2[2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2]} = D^{C^*}_{N,3P^N}(0) \end{aligned}$$

Substituting $K^{3P^N}_S$ into the expression for $D^{C^*}_{N,IP}$, we obtain

$$\begin{aligned} D^{C^*}_{N,IP}(K^{3P^N}_S) &= \frac{(2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2)\left[2(\beta_S+\gamma)\alpha_N + \gamma[\alpha_S + (\beta_S+\gamma)] - [2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2]c_N\right]}{4(\beta_S+\gamma)[4(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2]} \\ &\leq \frac{(\beta_N+\gamma)\left[2(\beta_S+\gamma)\alpha_N + \gamma[\alpha_S + (\beta_S+\gamma)] - [2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2]c_N\right]}{2[4(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2]} = D^{C^*}_{N,3P^N}(K^{3P^N}_S) \end{aligned}$$

Thus, we have shown that $D^{C^*}_{N,IP} = D^{C^*}_{N,3P^S} \leq D^{C^*}_{N,3P^N}$ in Region 1.

In Region 2, we require $D^{C^*}_{N,IP} = D^{C^*}_{N,3P^S} < D^{U^*}_{N,3P^N}$. At Region 2's leftmost boundary ($K_S = K^{3P^N}_S$), we have $D^{C^*}_{N,IP} = D^{C^*}_{N,3P^S} < D^{C^*}_{N,3P^N} = D^{U^*}_{N,3P^N}$. The national brand's demand under the $3P^N$, $D^{U^*}_{N,3P^N}$, is constant in K_S in Region 2, while the national brand's demand under $3P^S$ and IP continues to decrease in K_S. This implies that $D^{C^*}_{N,IP} = D^{C^*}_{N,3P^S} < D^{U^*}_{N,3P^N}$ in Region 2.

In Region 3, we require that $D^{C^*}_{N,IP} \leq D^{U^*}_{N,3P^S} \leq D^{U^*}_{N,3P^N}$. For $K_S \geq K^{3P^S}_S$, a direct comparison of $D^{U^*}_{N,3P^N}$ and $D^{U^*}_{N,3P^S}$ (see Tables 2 and 3 for the expressions, both of which are constants) reveals that $D^{U^*}_{N,3P^S} \leq D^{U^*}_{N,3P^N}$ because the two expressions share the same numerator, but the numerator for $D^{U^*}_{N,3P^S}$ is larger. We now turn to a comparison of $D^{C^*}_{N,IP}$ and $D^{U^*}_{N,3P^S}$. At the leftmost boundary in Region 3 ($K_S = K^{3P^S}_S$), we have

$$D^{C^*}_{N,IP}(K^{3P^S}_S) = \underbrace{\sqrt{\frac{2[(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2]}{2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2}}}_{\leq 1} \cdot \underbrace{\frac{2(\beta_S+\gamma)\alpha_N + \gamma[\alpha_S + (\beta_S+\gamma)] - [2(\beta_N+\gamma)(\beta_S+\gamma) - \gamma^2]c_N}{8(\beta_S+\gamma)}}_{=D^{U^*}_{N,3P^S}}$$

Thus, $D^{C^*}_{N,IP} \leq D^{U^*}_{N,3P^S}$ at the leftmost boundary in Region 3. Combining this result with the facts that $D^{C^*}_{N,IP}$ is decreasing in K_S in Region 3 while $D^{U^*}_{N,3P^S}$ is constant, we have that $D^{C^*}_{N,IP} \leq D^{U^*}_{N,3P^S} \leq D^{U^*}_{N,3P^N}$

in Region 3.

Finally, in Region 4, for $K_S \geq K_S^{IP}$, we have $D_{N,IP}^{C^*} = \dfrac{\alpha_N - (\beta_N+\gamma)c_N + \gamma c_s}{4}$. Multiplying both the numerator and denominator by $2(\beta_S+\gamma)$, the expression becomes $\frac{[2(\beta_S+\gamma)\alpha_N + 2(\beta_S+\gamma)\gamma c_S - 2(\beta_N+\gamma)(\beta_S+\gamma)c_N]}{8(\beta_S+\gamma)}$, which is less than or equal to $D_{N,3P^S}^{U^*}$ (cf. Table 3). Together with the fact that $D_{N,3P^S}^{U^*} \leq D_{N,3P^N}^{U^*}$ for $K_S \geq K_S^{3P^S}$, we have that $D_{N,IP}^{C^*} \leq D_{N,3P^S}^{U^*} \leq D_{N,3P^N}^{U^*}$ in Region 4.

Appendix I Third-Party Supplier Profits

In this appendix, we show that the third-party supplier prefers (profit-wise) that the national-brand supplier is the Stackelberg leader for K_S in Regions 1, 3, and 4. We conjecture that the result also holds in Region 2, although we have not been able to prove it because of the complicated form of $K_S^{3P^S}$. We have observed through numerous simulations that Region 2 covers a very narrow range of capacity levels (e.g., $K_S^{3P^S}$ is only a few percent larger than $K_S^{3P^N}$) and we have been unable to generate a counterexample to our conjecture.

The store-brand supplier's profits under the third party production configurations $(3P^N, 3P^S)$ for both the unconstrained and constrained equilibrium settings are given below:

$$\begin{aligned}
\Pi_{S,3P^N}^{U^*} &= \frac{(\beta_S+\gamma)\left[\gamma\alpha_N + (\beta_N+\gamma)(2\alpha_S+\gamma c_N) - \left(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right)c_S\right]^2}{2\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2} \\
\Pi_{S,3P^N}^{C^*} &= \frac{\left[\gamma\alpha_N + (\beta_N+\gamma)\left[2\alpha_S+\gamma c_N - 4K_S\right] - \left(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right)c_S\right]K_S}{2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2} \\
\Pi_{S,3P^S}^{U^*} &= \frac{\begin{array}{l}\left[2\gamma(\beta_S+\gamma)\alpha_N + \left(4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right)\alpha_S\right. \\ \qquad \left. + \left(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right)\gamma c_N - (\beta_S+\gamma)(4(\beta_N+\gamma)(\beta_S+\gamma)-3\gamma^2)c_S\right]^2\end{array}}{2(\beta_S+\gamma)\left[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right]^2} \\
\Pi_{S,3P^S}^{C^*} &= \frac{\begin{array}{l}\left[\gamma(\beta_S+\gamma)\alpha_N + (2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)(\alpha_S-2K_S)\right. \\ \qquad \left. +((\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)(\gamma c_N - 2(\beta_S+\gamma)c_S)\right]K_S\end{array}}{2(\beta_S+\gamma)((\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)}
\end{aligned}$$

To show that the third-party supplier always prefers the $3P^S$ to the $3P^N$ configuration in Regions 1, 3, and 4, we compare the applicable profits in each K_S region.

In Region 1, the store-brand factory has a binding capacity constraint under both $3P^S$ and $3P^N$. We can write $\Pi_{S,3P^S}^{C^*} - \Pi_{S,3P^N}^{C^*}$ as:

$$\Pi_{S,3P^S}^{C^*} - \Pi_{S,3P^N}^{C^*} = \frac{\gamma^3 K_S \cdot \left[(\beta_S+\gamma)\alpha_N + \gamma(\alpha_S - 2K_S) - \left((\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right)c_N\right]}{2(\beta_S+\gamma)\left[(\beta_N+\gamma)(\beta_S+\gamma)-\gamma\right]\left[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma\right]} \geq 0$$

This follows from (29), where we show the bracketed term in the numerator is positive.

We next compare the third-party supplier's profits in Regions 3 and 4, where the capacity constraint is not binding under either $3P^S$ or $3P^N$. The difference $\Pi_{S,3P^S}^{U^*} - \Pi_{S,3P^N}^{U^*}$ can be written as:

$$\Pi_{S,3P^S}^{U^*} - \Pi_{S,3P^N}^{U^*} = \frac{\begin{array}{c}\gamma^3 \cdot \left[(\gamma^2+\beta_S\gamma)c_S - (\gamma^2 + 2\beta_S\gamma + 2\beta_S\beta_N + 2\gamma\beta_N)c_N + (2\gamma+2\beta_S)\alpha_N + \gamma\alpha_S\right] \cdot \\ \left[\begin{array}{c}\left(8(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right)\left[2(\beta_S+\gamma)\gamma\alpha_N + (4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)\alpha_S + \right. \\ \left.(2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)\gamma c_N - (\beta_S+\gamma)(4(\beta_N+\gamma)(\beta_S+\gamma)-3\gamma^2)c_S\right] \\ -4\gamma^2((\beta_S+\gamma)\gamma\alpha_N + (\beta_N+\gamma)(\beta_S+\gamma)\alpha_S - (\beta_S+\gamma)((\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2)c_S)\end{array}\right]\end{array}}{32(\beta_S+\gamma)\left[2(\beta_N+\gamma)(\beta_S+\gamma)-\gamma\right]^2\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma\right]^2}$$

which we can show is decreasing in c_s. In Appendix D.1, we showed that (28) is an upper bound on c_s.

Substituting this upper bound into the above expression, we obtain a lower bound on the profit difference:

$$\Pi^{U^*}_{S,3P^S} - \Pi^{U^*}_{S,3P^N} \geq \frac{\gamma^6 \left[\gamma\alpha_S + (\beta_N+\gamma)\alpha_N - \left((\beta_N+\gamma)(\beta_S+\gamma)-\gamma^2\right)c_N\right]^2}{2(\beta_S+\gamma)\left[4(\beta_N+\gamma)(\beta_S+\gamma)-\gamma\right]^2\left[4(\beta_N+\gamma)(\beta_S+\gamma)-3\gamma\right]^2} \geq 0$$

Thus, the third-party supplier always prefers that the national brand is the Stackelberg leader for K_S in Regions 1, 3, and 4.